THE PHYSICAL COSMOLOGY
OF ALFRED NORTH WHITEHEAD

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INTRODUCTION AND ABSTRACT

Throughout the history of philosophy, cosmological theories have always deservedly enjoyed a position of special prominence. Of all recent cosmologies, or philosophies of Nature, perhaps the most comprehensive and satisfactory is that offered by Alfred North Whitehead.

Whitehead, always both mathematician and philosopher, enjoyed a full career as mathematician at Cambridge and London Universities before answering an invitation from Harvard University to a chair in philosophy there. His interests invariably carried him to the forefront of the advance, and his more technical mathematical works bore the imprint of a philosopher. His philosophy carried the marks of its birth in mathematics and the physical sciences.

Although his _Treatise on Universal Algebra_ (1898) won him an enviable reputation, it was his collaboration with Bertrand Russell in the first decade of the twentieth century on _Principia Mathematica_ which proved his pioneering genius. In the middle of this decade, Whitehead offered to the Royal Society of London a memoir entitled "On Mathematical Concepts of the Material World." This memoir, which fell into oblivion, employed the symbolic technique of _Principia Mathematica_ in solving the fundamental problem of importance to cosmological theory. Given a set of entities and a relation between those entities, Whitehead attempted to show the whole of Euclidean geometry to be an expression of the properties of the field of that relation. Certain extraneous relations served to associate the axioms with the material world of the physicists, of which Whitehead offered seven alternative concepts.

The first three volumes of Principia Mathematica had been published, and Whitehead had begun his work on the fourth, which was to have been concerned with the application of symbolic reasoning to the foundations of geometry and the problem of space. But by this time the scientific world had been captivated by the publication of the special and general theories of relativity by Einstein. These novelties naturally attracted Whitehead, who wrote several essays on the presuppositions of relativity. Whitehead was convinced that the principle and the method introduced by Einstein constituted a revolution in physical science, but found his explanation faulty.

A series of three important "Nature" volumes introduced the philosophy of "Nature" as conceived by Whitehead, using his own interpretation of the meaning of the new relativity. A powerful method of analysis, called the Method of Extensive Abstraction and having as its purpose the definition of spatial and temporal entities so as to avoid a circularity of reasoning was born at this period.

The third of the volumes was devoted entirely to the development of his own theory of relativity, to which the philosophically more satisfactory interpretation of relativity could be readily applied. From his original presuppositions Whitehead offered four alternative relativity theories, one of which coincided with Einstein's, and two of which were attempts at a unified field theory. The fourth, a theory of gravitation, used a physical element, the "impetus," instead of an infinitesimal metric element, as Einstein had done. This theory proved to be empirically less satisfactory than that of Einstein. But Professor George Temple generalized this fourth theory by using a space-time of positive uniform curvature, and results more satisfactory empirically than those of Einstein followed. The philosophical advantages of Whitehead's relativity were retained. This result seems to invite a more careful consideration of Temple's generalization of Whitehead's relativity than has been obtained at present.

But by this time Whitehead's speculations, which took as their restricted field the area of nature in which mind was irrelevant, began to concentrate on the enlarged field of cosmological theory in its points of contact with metaphysics. The most important discovery he believed he had made was that in this enlarged area, all the more special physical and extensive properties of nature were dependent for their existence upon process.

Now in his sixties, Whitehead accepted Harvard's invitation to a chair in philosophy. Within a very few years he returned to the United Kingdom to deliver the Gifford Lectures at the University of Edinburgh, in which the implications of adopting process as the central principle in the universe were systematically presented.

One outstanding feature of these lectures has been unfortunately ignored; it is a major and original suggestion of this thesis that the categoreal scheme of Process and Reality is really the axiomatic scheme of "On Mathematical Concepts of the Material World" generalized on the metaphysical level. An attempt at the application of the symbolic method to the axioms (categories of explanation and obligation) is made here. Thus the generalized problem in Process and Reality becomes, "Given a set of ontological existents and the operation of creativity, what axioms regarding the operation of creativity will have as their result that the more specialized discoveries of the humanities and the sciences follow from the properties of those entities forming the field of creativity?"

These lectures, although they offered a comprehensive metaphysical system justifying the operation of physical field theories, suffered under the misfortune that they were given at just the time when the quantum mechanics revolution was precipitated in the physical sciences. From the point of view of quantum mechanics, therefore, the philosophy of organism does not supply a satisfactory cosmology within which it can operate. This is especially unfortunate in view of his possibly superior physical theory of relativity; possible points of expansion to allow for quantum mechanics are indicated, although they do violence to the base of the philosophy of organism.

As the chief exemplification of the metaphysical principles, Whitehead postulated a brilliantly conceived metaphysical God who was important in physical cosmology. It is suggested that this metaphysical God is, nevertheless, inadequate to satisfy the demands of the religious conscience.

Despite the originality of most of the elements introduced by Whitehead, a full understanding of his meaning and an appreciation of his novelties is possible only by referring his writings to their proper settings. Thus, the philosophy of organism is explained against the background of the process philosophies of Bergson, Alexander, and Morgan. Because of its many similarities in respect to the setting of the cosmological problem and the essentials of the solution to the Timaeus, a special chapter is devoted to the correspondence between the two. Whitehead's relativity and philosophy of Nature requires an understanding of the development of the theory of relativity, the world-models of the relativistic cosmologies, and the attempts at a unified field theory. Similarly, the memoir of 1905 is described in a more general background setting forth a broad picture of the state of geometry, physical science, and philosophy at the turn of the century.

As a final reflection, certain presuppositions at the base of Whitehead's philosophy of organism are investigated and evaluated. The points believed by the present writer to be especially vulnerable in the philosophy of organism are exposed. An experiment in suggesting the prospectus of an alternative system which might avoid the difficulties, and incorporate the advantages of, the philosophy of organism, is made with the warning that it is no more than a suggestion.

Throughout the thesis, certain dominant strains of Whitehead's thinking can be detected: the importance in his mind of the axiomatic-deductive method in the sciences; the realization that prevalent habits of thinking need to be altered by new discoveries, but are resisted; the conviction that the sciences must be ontologically centered; the faith in field theories; and the conviction that cosmology must be the search for the forms in the facts; to designate the more outstanding convictions.

PART ONE

"ON MATHEMATICAL CONCEPTS OF THE MATERIAL WORLD"

CHAPTER ONE

BACKGROUND FOR "ON MATHEMATICAL CONCEPTS OF THE MATERIAL WORLD"

The stage on which Alfred North Whitehead's first essay in physical cosmology was to make its debut had been neatly prepared. Two issues of Nature[1] had announced that "On Mathematical Concepts of the Material World" would be presented at the first December meeting of the Royal Society of London in 1905. Geometricians already had arranged their axioms into consistent patterns, but had been experimenting with restating these axioms in terms of more primitive logical elements. Physicists, upon extending their mechanical ideas of the natural world, were confronted by paradoxes which suggested that a broader cosmology was imperative. Whitehead, whose reputation as the author of A Treatise on Universal Algebra had led to his election in 1903 to Fellowship in the Royal Society, had prepared a paper extending and surpassing the axiomatic researches of the

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1. 1905 "Diary of Societies" Nature, 73, 120. 1905 "Diary of Societies" Nature, 73, 144.

geometricians. Furthermore, he was prepared to suggest a physical interpretation of his geometry which would pass through the horns of the dilemma regarding action at a distance by use of a powerful application of the field concept.

Nature[2] carried a tardy notice that the memoir had been delivered; the preface of the memoir was published[3], and the entire memoir was printed in the London Philosophical Transactions.[4]

But there followed few immediate words of commentary or criticism[5], and later writers on similar subjects were apparently oblivious of the masterpiece that had been produced; Whitehead had failed to deflect the course of physics. Later, the memoir attracted some attention, and at least three men were convinced of the importance and merit of "On Mathematical Concepts of the Material World": Sir Edmund T. Whittaker[6], Victor Lowe[7], and W. Mays[8].

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2. 1906 "Societies and Academies: London: Royal Society" Nature, 73, 454.
3. 1906 Proceedings of the Royal Society of London, A, 77, 290-291.
4. 1906 Series A, 205, 465-525.
5. See Appendix B at the end of this Thesis.
6. 1948 "Alfred North Whitehead" Obituary Notices of Fellows of the Royal Society, 6, 284-285. Sir Edmund Whittaker has constantly emphasized the importance of this memoir in supervisory conferences during the preparation of this thesis.
7. 1941 "The Development of Whitehead's Philosophy" The Philosophy of Alfred North Whitehead, The Library of

A closer examination of the stage onto which the memoir emerged will help demonstrate the value of "On Mathematical Concepts of the Material World."

Mathematical Investigations in the Axioms of Geometry

The central problem which geometers were facing at the close of the nineteenth century was that of preparing a minimum set of basic definitions and axioms from which the rest of the theorems of geometry might be deduced. The whole point of minimizing the set of axioms is aptly suggested by Professor E. A. Milne's description of the operation as one of "deepening the axiomatic level."[9]

Having prepared a simple list of initial axioms, the geometer needed to demonstrate that these axioms were internally consistent. The set of axioms must similarly be tested to be certain that the axioms were primitive, *i. e.*, that no two or more might depend upon some earlier axiom

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Living Philosophers, *3*, 34-46.

8. 1950 "New Books: *Whitehead's Philosophy of Time*. By William W. Hammerschmidt" *Philosophy*, *25*, 180. The emphasis on the importance of the memoir for an understanding of Whitehead's later works was further expressed by Dr. Mays in a personal conference with him in Manchester on 27 May 1950.
9. 1944 "On the Nature of Universal Gravitation" *Monthly Notices of the Royal Astronomical Society*, *104*, 127.

for their validity. This search for the most basic statement of the axioms led to an investigation of the fundamental notions of logic, and to definitions of order and sequence of entities in certain arrangements.

Early in 1890 Mr. A. B. Kempe presented his "On the Relation between the Logical Theory of Classes and the Geometrical Theory of Points,"[10] wherein he introduced the notion of betweenness as a basic geometrical definition, an idea seconded in 1919 by Bertrand Russell.[11] "Betweenness" is a linear relation involving three distinct entities chosen from an infinity of homogeneous entities. Kempe suggested the symbolic form <u>ab•c</u> to denote this relation, which might be read, "Entity <u>c</u> lies between entities <u>a</u> and <u>b</u>." For the purposes of this thesis, this definition is the significant contribution of his memoir. The distribution of these linear triads throughout the system was defined by a set of four laws and a law of continuity. An additional sixth law assured that no two linear triads might have two non-equivalent entities in common. This sixth law implied the consequence that no two straight lines might intersect in more than one point, and excluded Riemannian geometry. Unfortunately, Mr. Kempe became in-

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10. <u>Proceedings of the London Mathematical Society</u>, <u>21</u>, 147-182.
11. <u>Introduction to Mathematical Philosophy</u>, 39.

volved with the case where $a = b$, and thereby diverted his attention from a more productive treatment of the subject. He suggested, however, that any geometrical interpretations might follow readily from his "Base System."

Henri Poincaré's famous La Science et l'Hypothèse (1901) introduced some statements over which controversy has raged. Geometrical axioms are only disguised definitions; they are not experimental facts.[12] These axioms must deal with homogeneous points in space, as contrasted with a non-homogeneous visual space where the points in the retina do not all perform the same function.[13] The issue of homogeneity later served as one of the main areas of dissention between the relativity writings of Whitehead and the orthodox Einsteinian relativists.

In December of the same year E. H. Moore presented to the American Mathematical Society a memoir, "On the Projective Axioms of Geometry,"[14] in which he contended it would be more desirable to use a finite linear segment as a basal element than to use a line of infinite length.[15] Such a procedure would imply a geometry founded upon spatial intuition rather than a set of axioms, and all its ensuing di-

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12. Page 66.
13. Ibid., 69.
14. 1901 Transactions of the American Mathematical Society, 3, 142-158.
15. Ibid., 144.

lemmas of apprehension. Apparently unwilling to attack the epistemological problems involved, Moore proceeded to offer (on the grounds of convenience) a set of seven projective axioms regarding lines and planes of infinite extension. At a later date, however, Whitehead attempted, in his "Method of Extensive Abstraction," to found his geometry on a more epistemologically satisfactory basis -- not a geometry of space, but of events.

In 1902 the first English translation of Professor David Hilbert's Göttingen lectures on The Foundations of Geometry appeared, and formed the definitive reference system for the axioms of Euclidean geometry. Hilbert offered the complete Euclidean system based on six groups of axioms including respectively (1) seven axioms of connection, (2) five axioms of order, (3) the parallel axiom, (4) the Archimedean axiom of continuity, (5) six axioms of congruence, and (6) an axiom of completeness. Each axiom was demonstrated to be independent of every other axiom by presenting a class of entities different from the points, lines, and planes of Euclidean geometry which satisfied all but the axiom in question, and which violated that axiom.

Oswald Veblen's "A System of Axioms for Geometry" followed in 1903,[16] and developed Kempe's work on betweenness to the point where it presented a system of eleven axioms and an axiom of continuity sufficient to produce Euclidean geometry. Veblen admitted that the non-Euclidean spatial notions of Bolyai, Lobatchewsky, Veronese, and others were equally suitable codifications,[17] although Poincaré had dismissed the conjecture as being only a verbal argument, and had suggested that all geometries could be transformed _inter se_.[18] It is from the system of twelve axioms offered by Veblen that Whitehead began his investigation. Veblen's axioms presupposed a class of homogeneous points and a relation of linear order; the only other assumptions being the validity of the laws of logical operations and counting. Unfortunately, Veblen's system labored under the defect that it dealt with an infinitude of static points, making the transition from geometry to the physical science of a changing universe difficult.

His treatment, however, indicated explicitly how the set of axioms might be altered to allow the possibility of non-Euclidean geometries. Using a technique similar to that of Hilbert, Veblen demonstrated each of the twelve axioms

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16. _Transactions of the American Mathematical Society_, _5_, 343-384.
17. _Ibid._, 343.
18. _Op. cit._, 66.

to be independent. If there be but one class of objects for which all the axioms hold, the system of axioms is called "categorical." A system wherein it is possible to add independent axioms is called "disjunctive." By omitting the parallel axiom, the Euclidean system becomes disjunctive, and admits the possibility of Lobatchewskian (hyperbolic) geometry.[19] Although Veblen did not indicate the full possible treatment of Riemannian geometry, the omission of the parallel axiom and one other[20] would produce a set of axioms harmonious with Riemann's elliptic geometry.

For three years Bertrand Russell had been engaged in preparing his The Principles of Mathematics,[21] a volume rich in thought about the logical foundations of mathematics and its relation to the material world. The major contribution of Russell's investigations to "On Mathematical Concepts of the Material World" was the suggestion of a method for incorporating the idea of change with the static geometrical notions.[22] Indeed, Whitehead directly borrowed this suggestion and offered it as "Concept II," in contrast with the static classical "Concept I" in his

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19. Veblen, op. cit., 347.
20. The axiom that a straight line can be infinitely extended.
21. Published in 1903.
22. Russell, The Principles of Mathematics, 468-469.

1905 memoir. For purposes of comparison with Whitehead's other "Concepts" and criticism, the exposition of the complete Concept II will be reserved until the second chapter of this discussion.

The question of the treatment of physical change in mathematical terms was, indeed, one of the advances of Russell's book. Change, he pronounced, must always involve at least two distinct entities (such as the points of space) and a triadic relation between those two entities and some of the instants of time.[23] The condition that matter cannot be either created or destroyed was expressed by the axiom that each particle of matter had the entire range of time-instants of a single time-series for its field. Thus, no particle of matter could have any segment in its life history when an instant of time was unrelated to it. The assumption that a single time-series is sufficient to describe the physical operation of the universe was not seriously challenged until the advent of the relativity era. The necessity of the existence of alternative time-series in the advance of nature received persistent emphasis at the hands of Whitehead in all his cosmological writings after the 1905 memoir.

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23. <u>Ibid.</u>, 469.

Russell was somewhat wary, however, of reiterating the dictum that no two material particles could occupy a single point at the same instant of time, and that one particle of matter cannot occupy two points of space at the same instant of time.[24] Yet, with respect to his suggestion that later became Whitehead's Concept II, he was careful to insure the very impenetrability of matter of which he was previously wary.[25]

With respect to the relation of betweenness, Russell then suggested that the current view[26] embodied sufficient conditions. The answer to the question as to whether the conditions were necessary was not so evident. There is the possibility that betweenness may be only a relation involving difference of vectorial sense.[27] Such a suggestion involves, however, an additional axiom relating difference of sense to sequential operations, as well as the additional definition of "sense." Whitehead did not include a consideration of this possibility in his 1905 memoir.

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24. Ibid., 467.
25. Ibid., 468.
26. As defined by Russell, betweenness is a relation of one term, y, to two others, x and z, which holds whenever x has to y, and y has to z, some relation which y does not have to x, nor z to y, nor z to x.
27. Ibid., 207-208.

Kempe's system had by this time reached Professor Josiah Royce, the American mathematician and philosopher. Attracted by the possibilities of expanding Kempe's betweenness relation to include the logical treatment of classes both in geometry and the exact sciences, Royce presented in 1905 his "The Relation of the Principles of Logic to the Foundations of Geometry."[28] Given a system (Σ) of homogeneous entities, Royce proceeded to demonstrate how the axioms of geometry might follow from the notion of betweenness. However, even betweenness is a derived notion, insisted Royce (in contrast to Kempe), and depends upon a more primitive relation called the O-relation. The O-relation is a polyadic relation which would express the manner in which any set of exhaustive, but, in their entirety, inconsistent, choices would stand to each other. Geometry then becomes the anatomy of a set of assertions that certain entities do, and certain others do not, stand to each other in the O-relation.[29] This enlarged definition of geometry compares favorably with those suggested at later dates by Whitehead. Aside from deriving geometry from the notion of the triadic betweenness rela-

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28. *Transactions of the American Mathematical Society*, *6*, 353-415.
29. *Ibid.*, 360.

tion, Royce's memoir diverges from that of Whitehead, who had received the American transactions only after his own memoir had been completed. O-relations and E-relations[30] were considered; the betweenness relation was examined[31]; and Royce then showed how Veblen's twelve axioms of Euclidean geometry might follow from his own postulated six principles governing the operation of O-collections. Royce's memoir is open to criticism as not accounting for the phenomena of a changing universe, as were all the previous treatments except that of Russell. Neither was the case of the null class considered. Royce's paper, too, suffered the fate of obscurity which befell "On Mathematical Concepts of the Material World."

At this level of development in the axiomatic treatment of geometry, Whitehead's 1905 memoir appeared. It was followed in quick succession by two short companion volumes in 1906 and 1907. The first, The Axioms of Projective Geometry, enjoyed a translation into French of its first chapter[32]; it is in this volume[33] that Whitehead defined geometry as "the science of cross-classification."

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30. E-relations are relations of entities which do not stand to each other in the O-relation.
31. Whereas Kempe used ab·c to mean that "c is between a and b," Royce used $\overline{F(c|ab)}$.
32. 1907 "Introduction logique a la géométrie" Revue de Métaphysique et de Morale, 15, 34-39.
33. Page 5.

A second novelty of the volumes is the consideration of the subject-matter as divided into projective and descriptive geometries, although Russell had previously suggested it.[34] Depending upon whether two coplanar lines necessarily intersect or not, the geometry is called projective or descriptive respectively.

A further innovation of Whitehead in the period involved in the production of the two tracts on geometry and the 1905 memoir was the assertion of the definitional nature of congruence. It must be assumed that Hans Reichenbach overlooked this contribution of Whitehead. Recently he said,"That the comparison of distances is also a matter of definition is known only to the expert of relativity. This result can also be formulated as the definitional character of congruence."[35] In _The Axioms of Descriptive Geometry_, Whitehead supported the argument that congruence can be mathematically defined.[36]

When considered as a unit, the two volumes on geometry may perhaps be thought of as the appendix promised for _A Treatise on Universal Algebra_[37], but which never ap-

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34. _The Principles of Mathematics_, 374.
35. 1949 "The Philosophical Significance of the Theory of Relativity" _Albert Einstein: Philosopher-Scientist_, _The Library of Living Philosophers_, _7_, 294.
36. Pages 44-48.
37. Page 32.

peared as such.

Veblen's "A System of Axioms for Geometry" had meanwhile attracted attention, and in 1909, A. R. Schweitzer proposed[38] the substitution of a dyadic relation[39] whose only usefulness was the elimination of the triangle transversal axiom. Schweitzer, however, realized that his system introduced an unnecessary complication in expressing the existence of a point order abc.[40] His paper ends on a note that is, without question, true of the systems thus far proposed: "... it does not seem possible to convert definitionally one system of axioms into the other and to retain that elegance which is peculiar to each individual standpoint."[41]

Once more the question of the independence of the axioms arose, and several treatments appeared with no novelties of importance for this thesis.

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38. "Note on a System of Axioms for Geometry" Transactions of the American Mathematical Society, 10, 309-314.
39. $\alpha\beta K\gamma\delta$. That is, α and β are in the same point-order as are γ and δ.
40. It became necessary to say (abc)R(abc): "a,b, and c are mutually related."
41. Ibid., 314.

Physical Concepts of the Material World

By the close of the nineteenth century the classical mechanical concept of the material world had reached its full flower. The corpuscular nature of matter had been almost conclusively established; the corpuscular nature of energy was discovered by Planck in 1900.

But within the classical concept there lay many unsolved problems. The exact nature of the material atom was still a matter of conjecture; its relation to the ether was the topic of much heated comment and apparent paradoxes. The discovery of radioactivity upset the static equilibrium of the classical concept and rendered more doubtful the questionable meaning of potential energy within that concept. Experimental results unexplainable except by _ad hoc_ hypotheses dotted the researches of the physical sciences. The unification of the corpuscular and the undulatory theories of light was still anticipated. Furthermore, the best of the theoretical physicists were urging a more stable groundwork for physics; among these, the names of Poincaré, Mach, and Poynting centered prominently in the years around the turn of the century.

Indeed, the haunting insinuation that physics needed a happier basis on which to found its researches infected many of the more creative physicists of the period. Nevertheless, the solutions offered were always couched in variations of the classical concept. Acknowledging Poincaré's criticism of the classical concept, an unsigned review in the Philosophical Magazine[42] suggested that perhaps Poincaré did not suggest a better alternative because of the impossibility of doing so. But Whitehead was prepared, in his "On Mathematical Concepts of the Material World," not only to codify the classical concept, but to offer several provocative alternatives to that theory.

The foundation of the mechanical theory of nature rested on the atomic hypothesis of matter and Descartes' mind-body dualism. Sir William Thomson (Lord Kelvin) suggested in 1867[43] that Helmholtz's discovery of the law of vortex motion in a perfect fluid led him to think that vortex rings immersed in the perfect fluid, ether, were the only true atoms. This vortex motion, the Wirbelbewegung, persists throughout the life history of the ring, and might serve as the repository for the unalterable distinguishing properties of matter. The presence of these unalterable properties supplied the only reason for the

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42. 1904 "Review of La Science et l'Hypothèse" Philosophical Magazine, (6), 7, 310.
43. "On Vortex Atoms" Philosophical Magazine, (4), 34, 15-24.

"monstrous assumption of infinitely strong and infinitely rigid pieces of matter."[44] Kelvin's subsequent discovery[45] that the equations for the propagation of laminous disturbances in a vortex-sponge are the same as the equations for the propagation of the light vibrations in the ether supported his theory. Indeed, the theory entertained a high degree of creditability for many years. The obvious difficulty of explaining how sensibly ponderous bodies might be derived from a rarely constituted ether did not distract the attention of many readers. Again, by assuming the properties of matter as qualities reposing in a substratum, Lord Kelvin implicitly subscribed to the Cartesian dualism. Whitehead formulated this concept mathematically in his 1905 memoir, but rejected it as being an unsatisfactory solution on the grounds of what was later to be his "fallacy of simple location." At a later date, he would have accused the theory of being an instance of the "bifurcation of Nature." Had the spin of the electron been discovered at this juncture, it is highly possible that the course of physics would have been deflected to a more serious consideration of the vortex-ring theory of

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44. Ibid., 15.
45. 1887 "On the Propagation of Laminar Motion through a turbulently moving Inviscid Liquid" Philosophical Magazine, (5), 24, 342-353.

matter as the basic solution to the problem.

Nevertheless, the "monstrous" corpuscular atomic theory advanced. Weber's[46] electronic theory of the electric current (as opposed to the fluid theory) was more amenable to precise scientific experimentation. By 1895 Sir Douglas Galton was prepared to declare that "electricity is closely connected with the vibrations which cause heat and light ... -- vibrations which may be termed the voice of the Creator calling to each atom ... to fall into its ordained position ... in the harmonious symphony which we call the universe."[47]

Of the more philosophically inclined physicists of this period, Professor Ernst Mach of the University of Prague was the one who perhaps most completely anticipated the later thoughts of Whitehead in an explicit manner. That everything in the physical universe, at least, is inextricably interrelated, and that any abstraction from this cooperation of the parts in order to study any given section of them must later be revised by reference to the whole, was a cēntral doctrine promulgated by Mach:

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46. Cf. E. T. Whittaker (knighted in 1945). 1910 _A History of the Theories of Aether and Electricity from the Age of Descartes to the Close of the Nineteenth Century_, 228.
47. 1895 "Inaugural Address by Sir Douglas Galton, K. C. B., D. C. L., F. R. S., President" _Nature_, _52_, 466, col. 2.

"Science can accomplish nothing by the consideration of individual facts; from time to time it must cast its glance at the world as a whole."[48] That such an interpretation continues to be considered valid in the scientific world is illustrated by the appeal made to it by Professor E. Finlay-Freundlich in a recent letter.[49] A further extension made by Mach of the necessary interrelationships of the parts of nature might easily have been included in any of Whitehead's metaphysical writings from 1925 until his death: "We must not forget that all things in the world are connected with one another and depend on one another, and that we ourselves and all our thoughts are also a part of nature."[50]

A declaration of a less fundamental metaphysical principle, although extremely important to his scientific theories was that "Absolute time is an idle metaphysical conception"[51] and an abstraction from the real nature of things. Again, this assertion was explicitly supported by Whitehead in his writings after the 1905 memoir, although in that paper absolute time was implicitly accepted. As with

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48. 1889 Die Mechanik in Ihrer Entwickelung, 433. Page references to this work refer to the second edition. The first edition appeared in 1883.
49. Letter from Professor E. Finlay-Freundlich to the author, dated 1950 August 28.
50. Mach, op. cit., 209.
51. Ibid., 209.

the notion of an inextricable interrelatedness in nature, Whitehead does not recognize the similarity of his own views and those of Mach in these two respects.

Experimental physics in 1898 received more evidence in support of the corpuscular theory of matter; in that year J. J. Thomson found a definite electrostatic charge associated with an ion produced by Röntgen rays passing through a gas.[52]

Six years later the same investigator developed an amazingly comprehensive mathematical explanation of the motions of certain numbers of negatively electrified corpuscles enclosed in a sphere of uniform positive electrification.[53] These electrons he supposed to be located at equal angular intervals around the circumference of a circle within the sphere. In the case of four electrons, there would be a steady state when those four electrons were at the corners of a square. In the case of six electrons, it would become necessary to have five electrons in a ring around the middle one. When the negative corpus-

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52. "On the Charge of Electricity carried by the Ions produced by Röntgen Rays" Philosophical Magazine, (5), 46, 528-545.
53. J. J. Thomson. 1904 "On the Structure of the Atom: an Investigation of the Stability and Periods of Oscillation of a number of Corpuscles arranged at equal intervals around the Circumference of a Circle; with Application of the results to the Theory of Atomic Structure" Philosophical Magazine, (6), 7, 237-265.

cles were not confined to one plane they would arrange themselves in a series of concentric shells. A comparison with the periodic table of chemical elements was effected, in which it was assumed that the atomic weight was the sum of the masses of the corpuscles contained in a given atom. Thomson's analysis had the further advantage of providing an explanation of radioactivity. When the angular velocity of the negative corpuscles decreased to reach a critical value, the system was no longer in equilibrium, and this might cause an ejection of a part of the atom, as in the case of radium. Since the inquiry was purely of a scientific nature, the metaphysical question of beginnings and endings in such a system was ignored. Furthermore, it would seem to be necessary to postulate a degenerative tendency on the part of nature; the appearance of new galaxies would be difficult to explain. The discussion, however, foreshadowed the postulation of the Rutherford atom in 1911, and represented a valuable scientific inquiry.

In the following year, 1905, J. Traube suggested[54] an alternative theory which subsequent research rejected. It was one in which the material atom had no need of positive or negative corpuscles, but whose chemical and electrical

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54. "On the Space occupied by Atoms: The Theories of Th. W. Richards and J. Traube" Philosophical Magazine, (6), 10, 340-352.

action was a function of the degree and placement of the contraction of the entire atomic corpuscle.

Frederick Soddy, reviewing "The Present Position of Radioactivity,"[55] implicitly accepted the Thomson hypothesis in talking of "the case of an electron revolving in an orbit within an atom."[56] He emphasized, however, that it was not yet known whether the electron itself possessed gravitational mass.

In the same year James H. Jeans offered a suggestion that the electron might be successfully interpreted by reference to the line-spectrum.[57] Indeed, that suggestion was ultimately brought to fruition by Niels Bohr in 1913 in connecting changes in the energy levels of the atoms with the spectrum of the respective elements.

The question of the constitution of the ether and its relation to material corpuscles was the second of the fields of investigation indicated for the end of the nineteenth century. In order to convey action from one material particle to a point in space it was deemed necessary to have an intervening medium as a vehicle; this was called the "ether." Michael Faraday in 1846 suggested[58]

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55. 1906 Nature, 73, 285-286.
56. Ibid., 285, col. 1.
57. 1906 "On the Constitution of the Atom" Philosophical Magazine, (6), 11, 605.
58. "Thoughts on Ray-vibrations" Philosophical Magazine, (3), 28, 348-349.

that the conception of an ether may be a meaningless abstraction. "I do not perceive," he wrote, "in any part of space, whether ... vacant or filled with matter, anything but forces and the lines in which they are extended."[59] He indeed carried this opinion to the extent of making a material atom the field of force surrounding a point-center. This treatment of matter as, in a sense, vectorial in nature, and associated with the lines of force, proved to be a challenge to Whitehead, although he made no reference to Faraday in the 1905 memoir. Faraday's suggestion, however, may easily have been the germinating influence on what proved to be the mainspring of Whitehead's cosmology. Given any material atom or event, and treating it as exerting a vectorial influence throughout the continuum in which it is located, that influence must necessarily interact with other similar influences from similar sources. The result would lead to a cosmology similar to that suggested earlier by Mach. Whitehead, moreover, avoided the difficulty which Faraday's thought introduced. For in a system such as Faraday's, how can the "stuffiness" of matter be explained?

A well-defined vortex-sponge model of the ether was

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59. Ibid., 348.

offered by Professor W. M. Hicks[60] in 1895. In this model cubical elements of fluid, each having a rotational circulation of its own, constituted the medium. Hicks, however, acknowledged the difficulty in explaining gravitation in the use of such a medium.[61] Such a model was also described in Whitehead's 1905 memoir, and dismissed in favor of one of his own formulation.

Larmor, challenged by the desire to account for the apparent existence of positive and negative charges on matter, postulated[62] many nuclei of intrinsic strain in the ether, known as "knots." Those knots twisted in one direction were positive; those in the other direction, negative. The temptation to draw a comparison of Larmor's "strains" with any interpretation placed on general relativity must, however, be resisted. By postulating structural differences within the ether, then, Larmor avoided the difficulty of a dualism between ponderous matter and

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60. 1895 "Opening Address by Prof. W. M. Hicks, M. A., D. Sc., F. R. S., President of the Section" Nature, 52, 472-477.
61. Ibid., 476-477.
62. 1894 "A Dynamical Theory of the Electric and Luminiferous Medium -- Part I" Philosophical Transactions of the Royal Society of London, A, 185, I, 719-822. 1895 "A Dynamical Theory of the Electric and Luminiferous Medium -- Part II" Philosophical Transactions of the Royal Society of London, A, 186, 695-743. 1897 "A Dynamical Theory of the Electric and Luminiferous Medium -- Part III" Philosophical Transactions of the Royal Society of London, A, 190, 205-300.

the ether, as did Lord Kelvin. Similarly, it encountered the same difficulty of explaining how matter, many times denser than the ether, might be satisfactorily explained in terms of the lighter substance. Furthermore, the "knots" explained only the presence of a dualism in the observed electrical phenomena associated with matter. A much more complex structure of the knots would be necessary to account for other physical qualities associated with material particles. Likewise, it appears that in such a cosmological system, there is a relevant metaphysical criticism. The natural universe would need to be of a tightly restricted form; novelties in nature would be difficult to explain. Such a universe would be a block universe, with its physical laws initially determined.

Mendeléeff pronounced his opinion upon the density of the troublesome entity, ether. Its substance must, he counseled,[63] possess an atomic weight of less than 5.3×10^{-11} in comparison with the standard of oxygen as 16.[64] Such a figure would be necessary in order that the ether

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63. 1904 "Prof. Mendeléeff on the Chemical Elements" Nature, 71, 65-66. The authorship of the article is unknown. In response to a letter suggesting a textual criticism, the author signed the acknowledgment only as "The Translator." (1904 December 1 Nature, 71, 102.
64. In the same paper, Mendeléeff postulated the existence of two other elements prior to hydrogen in the periodic table he so brilliantly fathered.

might escape the attractive force of the stars.

Perceiving the strained connection between matter and ether, J. H. Poynting, in his inaugural address to the Mathematical and Physical Section of the British Association,[65] expressed the hope that some day we would succeed in attributing to ether the work of gravitation. In terms of a space-time manifold around material bodies, and not in terms of ether, Einstein accomplished such a theoretical relation in 1915.

Sir Edmund Whittaker has concisely summarized the fate of the theories of the ether:

> However, most of the solid and liquid aethers of the nineteenth century had one feature which in the end proved fatal to them, namely, they were constituted of identifiable structural elements whose position could be traced from moment to moment, so that the phrase 'velocity relative to the aether' had a meaning, and consequently they were bound up with the principle that it is possible to define absolute velocity in space. With the advent of the theory of relativity in 1905, this principle was seen to be erroneous, and the search for a quasi-material aether came to an end.[66]

The discovery of radioactivity at the end of the nineteenth century posed another problem for the classical physicists. Rutherford in 1899 demonstrated[67] that there

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65. 1899 Nature, 60, 472, col. 2.
66. 1949 From Euclid to Eddington: A Study of Conceptions of the External World, 96.
67. "Uranium Radiation and the Electrical Conduction produced by it" Philosophical Magazine, (5), 47, 109-163.

were two types of rays emitted by uranium: an α-radiation, which is readily absorbed; and a β-radiation, which has great penetrating powers. Becquerel subsequently associated[68] the β-radiation with the discharge from a cathode-ray tube, and within a short time the α-radiations were identified with a charged helium atom.

Once again, Whitehead had prepared a concept of the material world which would admit of an interpretation of radioactivity, but the simultaneous publications of Einstein completely overshadowed Whitehead's contribution.

Potential energy is again a concept not readily woven into the classical mechanical treatment of nature. Points of space and instants of time cannot furnish a suitable frame of reference for that sort of physical quantity. Poynting expressed[69] a second hope that all the apparently distinct forms of energy (kinetic, potential, heat, light, chemical, etc.) might some day be satisfactorily related.

"On Mathematical Concepts of the Material World," in the opinion of this thesis, supplied the raw material for attacking each of these problems. Whitehead himself inspected the questions of the atomic nature of matter and its possible relation to any ether which might exist. For

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68. 1901 "Sur la radio-activité secondaire" Comptes Rendus, 132, 734-739.
69. Poynting, op. cit., 471, col. 2.

Whitehead, the notion of the existence of an ether at all was distasteful. Implicit within his system are potentialities for dealing with the other difficulties in the coherence of the physical theories. The implications of introducing additional definitions to account for these phenomena will be discussed in the second chapter of this thesis.

Philosophical Concepts of the Material World

Philosophical activity in the decade prior to "On Mathematical Concepts of the Material World" exhibited the advantages to be gained by any metaphysics which was willing to accept the increased authority of the natural sciences, and to gather their fruitful researches under its wings. The receptive fortresses of the various realistic schools were preparing forces with every scientific discovery to deal a staggering blow to cumbersome idealisms, and, by popular connotation, to all idealisms. Indeed, each bit of progress in understanding the material world seemed to give an implicit boost to those who felt that all metaphysical speculation was, by its very nature, a time-beguiling exercise especially adapted to an evening of armchair pastimes. Those years had a share of the philosophers who, curious though uninitiated in the scientific disciplines, drew righteous scorn from the initiated. But, however threatening in appearance to philosophical thinking scientific inquiry seemed, it was beginning to bear promise of assistance in giving insight into some of the paradoxes of its mother, philosophy. Moreover, it appeared that a thorough housecleaning in the philosophy of science was in order. Thus, these years were the beginning of the era, not yet finished, when both scientific theory and metaphysics were peculiarly prepared to derive tremendous benefit from a cooperative enterprise in the area of cosmology.

Probably the first philosophical inquiry of significance in this period was F. H. Bradley's Appearance and Reality.[70] The importance of this work lay, in the opinion of the present writer, not so much in the substantial positive contributions its pages contained, as in the fact that it precipitated a healthy discussion of the issues it involved, thereby accomplishing the author's avowed purpose. Adopting as his criterion of truth or reality the existence of non-contradictory experiences,[71] Mr. Bradley could indeed brand as "appearance," and not "reality," the phenomena of space, time, and even the "self." His two-edged sword of attack on space was used with equal diligence on the remainder of the "appearances" of our sense experience. Space cannot, he argued, be only a relation. Its parts are not relations, but extensive bits. The geometry of a triangle, for instance, describes a precise, observable area; its angles describe an identifiable quantity, the aperture between the including sides of the fig-

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70. The first edition appeared in 1893; the second followed in 1897.
71. Appearance and Reality, 136. Page references to this work throughout this thesis will be to the second edition.

ure. But, on the other hand, space can be nothing but a description of relations. For the very angles of the same triangle merely express the relationship between the including sides of any given angle; its area is nothing more than a function of the degree of translation of any two angles (another relation). And the triangle itself is no more than the relation between three non-collinear points. Hence the phenomena of space had been weighed against the feather of truth and found wanting.

Mr. F. C. S. Schiller, in explicitly attacking this technique of reduction [72] practiced by the "greatest of English sceptics,"[73] asserted that apparent inconsistency in observed phenomena by no means branded those phenomena to the death of ultimate non-existence. Schiller offered as the preferable alternative starting with "harmony" as the criterion of truth.[74]

It may, of course, be argued that in adopting such a criterion we may be violating the ultimate laws of the operation of nature, and thereby sentencing those harmonious speculations to final invalidity. Nature may be more or less a series of partly coherent and partly incoherent oc-

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72. 1903 "On Preserving Appearances" Mind, new series, 12, 341-354.
73. Ibid., 341.
74. Ibid., 344.

currences. The antithesis between coherence and incoherence has been hotly supported on both sides, with little success in the conversion of either side of the antagonists. The argument revolves around the validity of the opponent's criterion of truth, and final judgment in favor of either is wisely suspended. The advocates of coherence, however, generally emerge slightly better from such an argument, inasmuch as a stream of coherence usually is initially assumed by the advocates of incoherence.

Yet, to return to Mr. Bradley, an honest effort was made to reconstruct a meaningful account of human experience in terms of a network of internal relations within an Absolute. Treading close, however, upon the very ground he had previously condemned, Bradley asserted that "reality must own and cannot be less than appearance."[75] If the real have a character at all, it must be of such a nature as harmoniously to include everything phenomenal. Thus, Bradley introduced at this juncture the criterion of truth which Schiller had proposed. This harmony, with Bradley, must be effected through the Absolute, but this concordant structure is synthesized by "something beyond relations."[76] Such a harmony is not to be known directly

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75. Bradley, _op. cit._, 135.
76. _Ibid._, 214.

through mystical experience; that possibility had been slaughtered implicitly in the treatment of appearance.[77] It is likewise apparent that any of the usual categories of human thought are not applicable. To this attempted reconstruction, Schiller admonished, "The reality we have severed from its appearances we can never regain."[78]

What, then, could have been the cosmology implicit in Appearance and Reality? In what manner can the operations of nature proceed? The physical world itself, as a confusion of qualities and their relations, is mere appearance. And yet, it seems a scheme not brusquely discarded; it is "a necessary way of happening among our appearances."[79]

The distinctive essence of nature lies in its independence of the feelings and volitions of selves.[80] Here Bradley differed from Mach. Yet, Bradley followed by insisting that outside of finite personal experience there is indeed no natural world at all,[81] and "the addition of secondary qualities [to natural entities] ... in making Nature more concrete thereby makes it more real."[82]

Bradley's suggested approach for interpretation lay

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77. Ibid., 109.
78. Schiller, op. cit., 347.
79. Bradley, op. cit., 266.
80. Ibid., 268.
81. Ibid., 279.
82. Ibid., 493.

in the distinction of varying degrees of truth and reality. No matter what the judgment, it enjoys only a certain degree of validity -- an assertion dangerous to the very foundations of his own dialectic, and yet one favored by Hegel. Bertrand Russell has suggested that even John Dewey (perhaps unconsciously) assumes the validity of the doctrine of degrees of truth and reality.[83] The manner of attributing greater and lesser degrees of reality to phenomena rests completely on the crumbled remains of his appearance. "Other things being equal, whatever spreads more widely in space, or lasts longer in time, is therefore more real."[84] The laws of mathematics must also be less true, since they are more abstract and consequently more removed from concrete reality.

If a complete philosophy of world order were to be constructed on this scanty metaphysical groundwork, it would be that of postulating a universe of appearance against the background of the Absolute, the only wholly "real" standard by which judgments can be made. Relations within this system would then be of an internal kind, and there would be an emphasis upon "feeling" as a clue to their understanding. In respect to the emphasis upon

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83. 1946 A History of Western Philosophy, 851-852.
84. Bradley, op. cit., 370.

feeling, Whitehead agreed with Bradley, although there was considerable divergence elsewhere.

The world of "spirit" alone seems the only key to the solution of Bradley's enigma. Spirit is that quality or being most directly removed from lifeless nature. "Outside of spirit there is not, and there cannot be, any reality, and, the more that anything is spiritual, so much the more is it veritably real."[85]

It is difficult to decide into exactly what philosophical category Bradley could be placed as a result of Appearance and Reality. Materialism is explicitly excluded as a possibility. In the sense that realism may be denoted by postulating the independent existence of entities other than the human mind, Bradley is certainly that. Scepticism, although suggested by Schiller, seems to grant him hardly a fair trial. Although apparently all the usual paths of knowing something about the universe were closed, the very expression of a faith that there was some truth in human judgment in approximating reality excludes the sceptic's brand. The spirit of many of his statements, as exemplified in the preceding paragraph, tend definitely toward idealism. Bradley himself realized the difficulty of assigning any definite appraisal of his system in terms

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85. Ibid., 552.

of traditional schools of thought. His second engagement (with "Reality") was definitely both in the direction and spirit of absolute idealism.

Appearance and Reality was closely followed by the Gifford Lectures delivered by Dr. Alexander Campbell Fraser in 1894-1896. The series of lectures was designed to demonstrate the relative merits of theistic idealism as contrasted with pantheism, "panegoism," and especially materialism. Dr. Fraser attempted to demonstrate that the natural sciences could be more satisfactorily explained in the metaphysical framework of theistic idealism, or as he called it, "theism." The extent of the demonstration apparently consisted of (1) denying the possibility that conscious intelligence can be completely described in terms of material molecules or any other materialistic yardstick, and (2) suggesting that, on the other hand, a spiritual interpretation of all natural causative processes is as valid as any materialistic one. Actually a much stronger case for theism could have been constructed.

The entire series of lectures was infused with the idea that the "basis of human life is surely found in the faith that the ever-evolving universe is charged with meaning and purpose."[86] The purpose, of course, was given

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86. Fraser. Philosophy of Theism, 1, 242.

meaning through its connection with God.

The reasoning was in the typical theistic fashion: simple in style, moral in inclination, and buttressed by faith in God and His plan. Unfortunately, the theistic philosophers had still failed to produce a cosmology satisfactorily explaining the non-ideal portions of the universe.

In 1899 Professor Josiah Royce, the American mathematician and philosopher, whose work on the axioms of geometry has already been discussed,[87] was invited to offer a two-year series of the Gifford Lectures. Royce had been productive for some years in philosophy, and his position as a prominent absolute idealist was well established.

For the purposes of his 1899-1900 investigation, the relation of idea to external Being, Royce divided the philosophical field of investigation into four well-defined categories called respectively, realism, mysticism, critical rationalism, and idealism. Realism was here defined as the philosophical tradition that held that there was a fundamental divergence between idea and physical experience. More particularly, the essence of realism was the insistence of greater or lesser degrees of independence between idea and the experience to which it referred. The

87. Pages 11-12 of this thesis.

greater the emphasis on the independent existence of the two realms, the more decidedly realistic the doctrine. Mysticism was interpreted in the usual manner by making direct spiritual experience the criterion of truth. Critical rationalism is the name Royce applied to the school which suggested impersonal truths, such as energy, evolution, or the unconscious, as the explanation of reality. Ordinarily such a tradition could be included as a realist one, but for Royce it diverged from realism in that here the idea of external experience was intimately interrelated with the source of the experience itself. The fourth alternative was that of idealism, here treated as absolute idealism, and was a definitive formulation of the usual absolutist doctrines.

Attacking realism on the grounds that any independence of idea and its associated experience would eventually imply the total independence ofthe objects themselves, Royce insisted that such a standpoint was totally contrary to the facts of ordinary experience. "I distinctly decline to admit," he challenged, "that, in our concrete experience you can ever show me any two physically real objects which are so independent of each other that no change in one of them need correspond to any change of the other."[88] All things are inextricably interrelated by

88. Royce. The World and the Individual, 1, 125.

space, time, physical connections, and moral predicates. Total independence of objects in the external world would imply totally external relations and the postulation of an infinite series of intermediate external relations between any two relata, which even then could only at best roughly approximate the true relation between them.

William P. Montague replied to this criticism of realism[89] by drawing attention to the fact that no realist would admit that the hyper-independence which Royce attacked actually exists. The independence of the realist, he insisted, was not what makes an object real, but is only the means whereby our attention is drawn to its reality. Furthermore, this small amount of independence would by no means implicitly require totally external relations. Apparently Montague has found the real difficulty with Royce's argument against realism -- an improper appraisal of his adversary's actual assertions. It is for the very reasons which Royce has given that the realists do not inhabit that extreme position which lies beyond realism.

Critical rationalism, whose spiritual fatherhood Royce ascribed to Immanuel Kant,[90] was described as the ontological interpretation which was current in the nineteenth century, a period described as philosophically bar-

89. 1902 "Professor Royce's Refutation of Realism" The Philosophical Review, 11, 43-55.
90. Royce. Op. cit., 205.

ren by Professor Ritchie.[91] One of the few technical scientific speculations Professor Royce discussed soon bore fruit opposite to that which he had prédicted: only the following year Planck established the corpuscular nature of energy.[92] With respect to the relations between pure mathematics and physics, Royce declared their logical independence early in the lectures.[93] Such a statement was not only alien to the decision of Poincaré, who maintained that without material bodies in nature there would be no geometry,[94] but also to the final conclusion of Whitehead's "On Mathematical Concepts of the Material World."

In developing his own system of absolute idealism, Royce admitted, with the mystics, that whatever is must somehow be One. The mystical conception, however, fails to express sensed physical reality sufficiently well, and so absolute idealism must be tested for adequacy.

Whatever is real must as such, Royce maintained, be the complete individual embodiment of the internal meaning

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91. 1950 November 14. Lecture to Senior Honours students, Edinburgh University.
92. Royce had declared, op. cit., 241: "But nobody of any authority, I suppose, is yet prepared to maintain in any decisive way that the energy of the physical world consists of a collection of ultimate individual units or bits of energy, which retain their individual identity, and as individuals transfer themselves from one part of matter to another."
93. Ibid., 9.
94. La Science et l'Hypothèse, 80.

of finite ideas. These complete embodiments may be known, not only by our own finite consciousnesses at any given moment, but constantly by a consciousness inclusive of our own, that of the Absolute. Thus the fundamental structure of the entire universe must be both teleological and conscious.

Such a system of absolute idealism, although firmly knit, must still be subject to the powerful counter-charge that it relegates to the Absolute, and hence to a region imperfectly known to finite conscious selves, an interaction between mind and body which is apparently dualistic. Further, it accounts for the seemingly independent existences of finite human souls in a remote manner. Further yet, the necessity for considering the material universe to be conscious had not been proved, but only postulated.

One of the technical problems precipitated anew by the scientific law of the conservation of energy was that of the interaction between mind and body. At this stage of its development, the relation of the mind-body problem to cosmology was extremely restricted. Whitehead's later works, however, attempted to solve the problem as a logical consequence of the cosmology postulated.

It appeared that, in order not to violate this conservation of energy principle, a direct interaction between a substantially contrasted mind and body would be impossible. Leon M. Solomons[95] followed by Frank Thilly[96] held that interactionism was indeed possible without violating scientific principles. Some interactionists argued that there might well be a mental energy analogous to physical energy, and that the principle of the conservation of energy would then extend over a psycho-physical world as easily as over an exclusively physical realm. Other interactionists, such as Rehmke and Wentscher, held that extra-physical causes might be construed as releasing an undetected potential energy in the physical world.[97] Thilly himself was of the opinion that the physicists should be taken at their word in defining the principle to be valid in the material universe, and not extended without experimental evidence to the mental realm.

But the obvious alternative of parallelism was no more attractive. If mental processes be assumed to operate independently of physical processes, there is the necessity for accounting for the empirical evidence providing a positive correlation between the two. An attempt to

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95. 1899 "The Alleged Proof of Parallelism from the Conservation of Energy" The Philosophical Review, 8, 146-165.
96. 1901 "The Theory of Interaction" The Philosophical Review, 10, 124-138.
97. Ibid., 129.

codify this correlation by using the total external stimulus and the least sensible difference as logarithmically related elements was inaugurated by Weber and improved upon by Fechner. This has been denominated the Weber-Fechner Law in psychophysics.

Professor Ritchie, reviewing the value of the Weber-Fechner Law for the British Association in 1949, pointed to the fact that the law "states the relation between two physical quantities, namely total stimulus and least distinguishable increment of stimulus; the 'psycho' part is assumed to be parallel, not found to be so."[98]

Robert Arnold conceived the solution of the mind-body problem in a new light, namely, that certain parts of the brain must act as external to mind.[99] Such a solution, however, merely transferred the problem to another area and in no way offered a clue to resolving the difficulty.

Whitehead himself later implicitly acknowledged the interactionist hypothesis to be the better, and the central problem of explaining how the interaction occurs was a direct consequence of his cosmology. The mind-body problem was ignored in the 1905 memoir, however.

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98. 1950 "The Relation of Body and Mind: Symposium: The Relation of Brain to Mind" _The Advancement of Science_, _7_, 50.
99. 1904 _Scientific Fact and Metaphysical Reality_, 168.

The traditional cosmological problem of the nature of time had begun to assume a new importance in philosophy, and this was accelerated with the advent of relativity theory. The status of time in relativistic cosmology will, however, be postponed to a later chapter.

Walter Smith, in speaking of "The Metaphysics of Time"[100] in 1902, emphasized the Bradleian doctrine of degrees of reality with respect to time: the present is real, the past and the future are unreal. Yet the time-concept embraces the idea of change as its most distinctive character. The outstanding aspect of Smith's treatment is the attributing of a spatial character to time; when a succession of events is thought of, they are arranged in a spatial order. This confused manner of thinking about time was not original with Smith, and was a favorite assertion with Bergson. The second notable aspect of Smith's treatment is that a given individual at one instant becomes another distinct individual at a later instant. In a sense, this became a statement about Whitehead's actual entities, but the statement must not be taken too literally. For Whitehead there was a new actual entity, but something of the antecedent entity was preserved in the new one. For Smith it was only in an absolute consciousness that these

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100. 1902 The Philosophical Review, 11, 372-391.

various individual-instants coexist and are interrelated.[101] The question of the continuity of a given individual from instant to instant was by no means satisfactorily considered.

The following year Smith reiterated the Kantian notion of the subjectivity of space,[102] and further asserted its evolution.[103] Such an evolution is exemplified in the increasing natural perfection of the visual sense, and in the increasing refinement of abstract ideas.

Bertrand Russell had also investigated the idea of spatial arrangement,[104] and formulated both the absolute and relational theories. Time, on the other hand, must be of an absolute nature, and can be represented by a single one-dimensional series of instants.[105] At a later date, Russell had been influenced by Whitehead to abandon not only instants of time as ultimate, but points of space and particles of matter as well, and to substitute for them logical constructions composed of events.[106] It is indeed doubtful, as William James has suggested, whether an-

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101. Ibid., 387.
102. 1903 "The Idea of Space" The Philosophical Review, 12, 493-510.
103. Ibid., 499.
104. 1901 "Is Position in Time and Space Absolute or Relative?" Mind, new series, 10, 293-317.
105. Russell, The Principles of Mathematics, 468.
106. 1937 The Principles of Mathematics, second edition, xi.

thing such as an instant of time is a component of direct human experience.

In France, Henri Bergson had been, for over a decade, introducing a type of philosophy known as a "process" philosophy, destined to gather greater momentum in his later writings and to attract a wide circle of adherents. The _Revue de Métaphysique et de Morale_ of 1903 bore an article by M. Bergson[107] which codified his metaphysical background of this new philosophy. The dependence of this article upon _Matière et Mémoire_ and the _Essai sur les Données Immédiates de la Conscience_ is apparent. The duty of metaphysics, thus conceived, was to demonstrate the nature of reality as it might be known, not through intellectual analysis, but through a direct intuition regarding its composition. As an example, the one reality directly known to us is that of our own personality flowing continuously through temporal durations. Bergson held that an intellectual analysis of this reality would fail to describe it adequately.

Instantaneous position in space is a mental abstraction; a moving body is never really in any one of the points along its path. The most that can be said is that

107. "Introduction a la Métaphysique" _Revue de Métaphysique et de Morale_, _11_, 1-36.

the body passes through them.

All reality can be characterized by one predicate: it is constantly changing. Indeed, the inner life of a human being progresses according to a unity of direction. It is because of the similarity of this metaphysical character of reality with Bergson to that of the philosophy of organism of Whitehead that only a passing summary of Bergson's cosmology is given in this chapter. A fuller discussion will appear in Chapter VII.

Bergson's metaphysical system was one of those not investigated implicitly by Whitehead's "On Mathematical Concepts of the Material World." Curiously enough, Whitehead himself was to develop his later philosophy into one which had many points of similarity with that of Bergson. How such a cosmology developed in Whitehead will be discussed in later chapters.

Robert B. Arnold, in an attempt to effect a liaison between science and idealistic metaphysics, emphasized not a dual nature of the universe, but a triple one, composed of matter, ether, and mind.[108] The ether was conceived to represent an existent necessary background for individual material entities. The two, however, because of their close relationship, might be considered as of a single na-

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108. 1904 Scientific Fact and Metaphysical Reality, 235-261.

ture for purposes of contrast with mind.[109] Nature itself was "atomically active,"[110] but in its material aspect did not engage in a teleologically unified process.

Thus, in 1905, mathematics was well prepared to receive a precise statement of the direct relations of its own pure discipline to that of physics. Physics, having developed the classical theory to its mature embodiment, had observed phenomena unexplained within the framework of that theory. These phenomena, as well as some internal paradoxes in the classical system, made it evident that further sizeable advances might follow a more inclusive theoretical statement of cosmology. Philosophy had been repeating its age-old metaphysical battles, sometimes incorporating, many times omitting (for science had become highly specialized), the suggestions of natural science. A cosmological scheme satisfactorily explaining these scientific suggestions could well have changed the growth of both natural science and philosophy for ages to come. Whitehead's "On Mathematical Concepts of the Material World," although it promised these improvements, failed to produce any of these.

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109. Ibid., 252.
110. Ibid., 164.

BIBLIOGRAPHY

CHAPTER ONE

Items appear in the Bibliography for the Chapter or Chapters in which they are specifically mentioned. Because of the length of the Bibliography, one or more index letters appear after the item entry. The interpretation to be placed upon these index letters is

(A) "Auxiliary" -- The item contains a discussion of important points which are not, however, the main topic of the chapter.
(B) "Background" -- The item gives a valuable general discussion of various points in the chapter, or is valuable as a historical background.
(C) "Central" -- The item represents the central point of the discussion in the chapter, or a thorough consideration of that problem. This index designation will be assigned sparingly, and the (A) class will contain items which are doubtful applicants for (C).
(D) "Documentation" -- The item has been used in the chapter solely for documentation or to present minor points.

Any assignment of an index letter refers only to the use of that item for the chapter immediately at hand. It in no way reflects the opinion of the writer of this thesis upon the value of the item in itself or in other contexts. It does not reflect any judgment whatever upon the author's ability to discuss pertinent points, but only to the actual use that item has enjoyed in the development of the pertinent chapter. Thus, the same item may have different designations in different chapter bibliographies.

ANONYMOUS. 1905 "Diary of Societies" Nature, 73, 120, 144. (D)

ANONYMOUS. 1904 "Prof. Mendeléeff on the Chemical Elements" Nature, 71, 65-66. (D)

ANONYMOUS. 1904 "Review: La Science et l'Hypothèse" Philosophical Magazine, (6), 7, 310. (D)

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CHAPTER TWO

"ON MATHEMATICAL CONCEPTS OF THE MATERIAL WORLD"

Whitehead's "On Mathematical Concepts of the Material World" was submitted to the Royal Society of London, and was an essay in cosmology in so far as it defined certain relationships among certain postulated entities in an attempt to explain the operation of natural phenomena. Although the author explicitly asserted that the interest of the paper was primarily logical,[1] here equated with mathematical, he exhibited certain cosmological preferences which later were to develop into significant portions of his philosophy. Special techniques in the analysis of physical laws and mathematical relations exhibited in this memoir later became the salient features of his mature cosmology. Both Sir Edmund Whittaker[2] and Dr. Victor

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1. Alfred North Whitehead. 1906 "On Mathematical Concepts of the Material World" Philosophical Transactions of the Royal Society of London, A, 205, 465. For ease of reference, this memoir will hereafter be designated MCMW in footnotes.
2. Sir Edmund T. Whittaker, 1948 "Alfred North Whitehead" Obituary Notices of Fellows of the Royal Society, 6, 281-296. This point has been consistently emphasized in supervisory conferences by Sir Edmund Whittaker throughout the preparation of this thesis.

Lowe[3] have emphasized the indispensability of the 1905 memoir to an understanding of Whitehead's later writings.

It will be a major assertion of this thesis that not only is a knowledge of "On Mathematical Concepts of the Material World" indispensable to a satisfactory understanding of the cosmology of Whitehead, but that it is an ancestor in a direct line of the relativity writings of Whitehead and of Process and Reality. There have been apparently no publications giving a thorough consideration to this important assertion, and Sir Edmund Whittaker, Dr. Victor Lowe, and Dr. W. Mays have been the only persons of whom this author is aware who have realized the importance of the 1905 memoir. Because this thesis is being developed chronologically, the major part of the documentation of the assertion will occur in Chapters V, VI, VIII, IX, and X. This chapter will be devoted to an exposition of the memoir, an indication of the parts which will assume later importance, and an assessment of the value of "On Mathematical Concepts of the Material World."

If, Whitehead suggested, physical science in the future might be governed by a unified principle more inclu-

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3. 1941 "The Development of Whitehead's Philosophy" The Philosophy of Alfred North Whitehead, The Library of Living Philosophers, 3, 33-46.

sive than that which at that time was prevalent, the memoir might have added direct bearing on physical researches. It is the opinion of this thesis that the direct applicability of the precise findings of the memoir has passed the time of its potential usefulness. However, it is asserted that the method of "On Mathematical Concepts of the Material World" is highly pertinent to physics and to philosophy of the present and the immediate future. It is also suggested that the memoir could be profitably studied by both physicists and philosophers as a methodological model for their own speculations. More especially, inasmuch as it is asserted that the memoir is indispensable to a full understanding of Whitehead's later works, it still has a pertinent message by way of providing increased understanding of Whitehead's final philosophy. In short, it is suggested that "On Mathematical Concepts of the Material World" has suffered a totally unwarranted obscurity.

The 1905 memoir was devoted to an extended consideration of seven ways of interpreting the material universe. The last of these seven proved to be so rich in suggestions that the interpretation developed was only an outline of one possible case. The difficulties of expanding the possibilities lay in the correct choice of definitions regarding physical properties associated with elementary components of the physical world, and science had not yet provided suitable raw material for application to the concept.

One of the primary reasons for the failure of the memoir was the relative obscurity of a systematic notation for logical processes of thought. The first volume of Principia Mathematica was not to appear for several years,[4] and the reading of series upon series of unfamiliar symbols discouraged many who would be best qualified to suggest further developments to the cosmology. For "On Mathematical Concepts of the Material World" was developed primarily in terms of the symbolism introduced by Peano and improved by Whitehead and Bertrand Russell. The bulk of the clear text exposition is little more than an approximate transfer into verbal statements of the more precise findings of the symbolic statements.

For ease of transformation of statements and direct reading of those statements, the Whitehead symbolism was far superior to any used by Kempe, Moore, Veblen, or Royce. Veblen's statement of the relation of "betweenness" approximated Whitehead's codification,[5] but was not developed beyond that simple statement.

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4. Volume I appeared in 1910.
5. Where Veblen used (abc) to express the relation "a, b, and c are in that linear order," Whitehead emphasized the relational aspect by using R;(abc).

The paper considers a set of entities, homogeneous in nature, although not explicitly delineated as such; both Kempe and Royce have underscored the necessity for this presupposition of homogeneity. In later writings on relativity themes, Whitehead repeatedly emphasized the necessity of spatio-temporal homogeneity for a rational cosmology. These entities need not be the points of Euclidean space, nor the material particles of classical physics. Indeed, in later chapters several alternative entities not considered by Whitehead, will be suggested as appropriate candidates. When, however, the entities consist of the points of Euclidean space, the problem receives its simplest form. Whitehead ignored a treatment using number sets as entities, although had he proved the independence of his axioms, would certainly have been forced to do so, as did Veblen, for example. Instead, Whitehead found the use of infinitely long rectilinear extensions the most fruitful for cosmology.

These homogeneous entities form the "field" of a certain polyadic (not necessarily triadic) relation called "R." Indeed, the classical treatment requires that R be triadic for an expression of betweenness, but cases were developed by Whitehead where the terms of the relation R numbered four and five.

Respecting the logical implications of these relations, certain axioms can be formulated. The problem of the memoir is, "Given a set of entities which form the field of a certain polyadic ... relation R, what 'axioms' satisfied by R have as their consequence, that the theorems of Euclidean geometry are the expression of certain properties of the field of R?"[6]

It is the suggestion of this thesis that the basic problems of cosmology can be expressed in words highly similar to this explicit problem of Whitehead: Given a set of entities which form the fields of certain polyadic relations R, what axioms satisfied by R have as their consequence, that the postulated physical relationships are the expression of certain properties of the field of R? Derivatively, What is the nature of the entities, and What is the nature of R? also become intensely important to cosmological research. This formulation, of course, presupposes that an axiomatic, as contrasted with an inductive, procedure is being used. Throughout his writings, Whitehead showed a preference for the axiomatic method, and documented the results of that method with the indications suggested by induction. The appraisal of the relative merits of the inductive and axiomatic methods will be made in later pages of this thesis.

Throughout Whitehead's memoir only three-dimensional

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6. MCMW, 465.

Euclidean space is considered; but non-Euclidean geometry, a favorite study of Whitehead, could admittedly have been erected upon a similar superstructure. This could be accomplished by rendering the system of axioms disjunctive, as Veblen had suggested, or the properties of the fundamental relation might be varied.

The author, intent upon producing an interrelation between a universe of change and a spatial treatment of its entities, specified that the notion of time was indispensable. This sentence, in its entirety, might be used to characterize any of Whitehead's later works; there is an obvious debt to the 1905 memoir on this score, as on the use of the axiomatic method.

Time must be conceived as a single dyadic serial relation, and must be included in the consideration of each concept. The entities forming the field of the time-relation must be homogeneous instants of time. This consideration of time as a succession of instants is not a characteristic of Whitehead's later writings. The 1905 memoir is indeed the last place where Whitehead uses a single time-series to describe the advance of nature.

The "material world" suggested in the title is defined to mean the set of relations and the entities which form the field of these relations. Here, for the first time, is a characteristic of Whitehead's later philosophy, the consideration of both the entities and the relations between them as being the universe. Hence, in Process and Reality, it is not just the actual entities and the eternal objects which make up the universe, but also the prehensions, the propositions, the nexūs, and the contrasts. Within this material world there are relations called "fundamental relations," characterized by the fact that they are relations not defined in terms of alien entities, such as, for example, the percipient of an entity. Indeed, this specific relation of perceiver to perceived is expressly denied as being of concern to the memoir.[7] There is a complete absence of epistemological consideration with respect to the precise development of the concepts. Epistemological difficulties are suggested as powerful enough reasons to reject certain concepts, but the concept itself is developed independently of any percipient.

The hypotheses respecting the functions of the fundamental relations are called "axioms" of that concept of the material world. Whitehead was even now carefully defining words in a technical sense which conveyed a precise meaning not ordinarily agreed to be the specific denotation of the word.

Thus, the full meaning of a "concept of the material world" is threefold: (1) the complete set of axioms, (2)

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7. MCMW, 467.

the appropriate definitions of entities and the relations between them, and (3) the propositions resulting from the interaction between the axioms and the definitions.

"Ultimate existents are defined to be the complete class of those entities which are members of the fields of the fundamental relations. Again, Whitehead avoided both an ontological and an epistemological bias in the memoir by denying any relation between his "ultimate existents" and those of the metaphysicians. It is necessary that instants of time be included among the ultimate existents of every concept. Despite the fact that Whitehead carefully excluded metaphysical implications from being attached to his ultimate existents, they nevertheless form the basis for the cosmology appropriate to any concept. The ultimate existents ofthe 1905 memoir became derived entities, rather than self-subsistent ones, in his relativity writings, and remained so in all the later writings.

When the class of ultimate existents is considered exclusive of the instants of time, the resulting class is to be known as the class of "objective reals." Again, there is to be no metaphysical connotation to be attached to such an entity. The distinction between ultimate existents and objective reals is a novelty of Whitehead's thought, although perhaps foreshadowed by Bertrand Russell.[8]

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8. 1903 The Principles of Mathematics, 468.

Possible applicants for inclusion in the class of objective reals vary with each concept; they may be points of space, particles of matter, or[9] linear objective reals -- the linear extensions considered as a unit. They may be combinations either of points and particles or[10] of linear objective reals and corpuscles. In no case are all three applicants members of the class of objective reals in the concept. When two classes of objective reals are necessary for a concept, that concept will be called "dualistic"; when one class only is the class of objective reals, the concept will be called "monistic." "Occam's razor," continued Whitehead, "formulated an instinctive preference for a monistic as against a dualistic concept."[11]

There are two senses in which Occam's razor may be considered with reference to the 1905 memoir, one valid and one invalid. In the first sense, Whitehead had prepared a paper considering the mathematical concepts of the material world. In order to forge a more unified mathematical concept, it would be highly desirable to reduce, if possible, the class of objective reals to one class. Here the application of Occam's razor would be valid, for there would be no limiting factors other than the necessi-

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9. This "or" is to be considered in the exclusive sense.
10. "Or" in the exclusive sense.
11. MCMW, 468.

ty for mathematical rigor.

In the second sense, the memoir was concerned with mathematical concepts of the material world. The researches of physical science into the entities composing the physical world by no means indicated that there should be but one class of objective reals. Indeed, despite hopes (similar to those of Poynting[12]) of a unified material "ultimate," there was no near-conclusive evidence that such was the case. Physical science could by no means allow Occam's razor to remove any of its entities at that time. In this sense, Whitehead's application of Occam's razor to the 1905 memoir is metaphysically questionable, and empirically invalid.

There is evidence to believe that Whitehead was using Occam's razor in both senses. Possibly he meant it in the first sense, for the mathematical consideration in this memoir logically preceded the physical applications. The concluding statement of the memoir leads to the belief that he was using it in the second sense as well:

> In regard to the simplification of the preceding axioms, ... the ideal to be aimed at would be to deduce some or all of them from more general axioms which would embrace the laws of physics. Thus these laws should not presuppose geometry, but create it.[13]

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12. 1899 "Opening Address to Section A (Mathematics and Physics) of the British Association" Nature, 60, 470-474.
13. MCMW, 525.

In such a case, it would be the task of experimental physics, and not of Occam's razor, to decide upon the number of objective reals necessary for any given mathematical concept of the material world. Unfortunately, there has been no discussion of this ambiguous usage either in the memoir itself or in any of the commentaries known to this author. The suggested fate of the principle of Occam's razor in connection with the 1905 memoir is that it be dismissed entirely, and that the closing sentences of the memoir be considered as a more authoritative suggestion as to what number of classes shall compose the class of objective reals. A philosophically more mature consideration of the problem appeared in Whitehead's writings of the relativity era.

The class of fundamental relations is, in all concepts, exhausted by three mutually exclusive subclasses of relations: the time relation, the "essential relation," and any "extraneous" relations which may be integral to that concept.

The essential relation is a single polyadic relation, based on the idea of betweenness for the first four concepts, and on a defined relation of "order" in the last three. From the essential relation alone, all the propositions of Euclidean geometry were shown to follow, although other class relationships are implied as well. It is in the sense that geometry is derived from a triadic relation of betweenness in the first concept that Whitehead's paper is similar to those of Kempe, Veblen, and Royce. The field of the essential relation varies with each concept, but may consist of all or part of the class of ultimate existents.

The extraneous relations may number one or be of an indefinitely large number, depending on whether they are used to determine "kinetic axes" of reference for the determination of velocity or to locate the particles in space respectively. Kinetic axes will be discussed at the point where they are necessary for the concept.

Because geometry had always been construed in the sense of the first concept, the classical dualistic concept (the objective reals are points of space and particles of matter), a new definition of geometrical propositions was needed for the purposes of the 1905 memoir. "A proposition of geometry is any proposition (1) concerning the essential relation; (2) involving one, and only one, instant of time; (3) true for any instant of time."[14] Thus, because condition (1) gave classical geometry an enlarged field of operation, many propositions involving classes and their relations became possible which do not ordinarily qualify as "propositions of geometry." It is

14. MCMW, 469.

the opinion of this thesis that this definition of geometry in the 1905 memoir is the immediate ancestor of Whitehead's more notorious later definitions of geometry as "the science of cross-classification"[15] and as "the morphology of nexūs."[16] It is further suggested that the less notorious, but equally suggestive, definitions of geometry as "the doctrine of loci of intermediaries imposing perspective in the process of inheritance"[17] and as "one chapter of the doctrine of Pattern"[18] were likewise fathered in the mentioned passage in the 1905 memoir. These notions are similar in tone to Royce's statement that geometry is the study of the implications of assertions that entities do, or do not, stand to each other in the O-relation.[19]

The Seven Mathematical Concepts of the Material World

Of these seven concepts, the first four consider points of space as the basic geometrical entities; the

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15. 1906 The Axioms of Projective Geometry, 5.
16. 1929 Process and Reality, 461.
17. 1932 "Objects and Subjects" Philosophical Review, 41, 143.
18. 1941 "Mathematics and the Good" The Philosophy of Alfred North Whitehead, The Library of Living Philosophers, 671.
19. 1905 "The Relation of the Principles of Logic to the Foundations of Geometry," Transactions of the American Mathematical Society, 6, 360. See also pages 11-12 of this thesis.

last three, linear objective reals. Accordingly, they are named "punctual" or "linear," as the case may demand.

Concept I, the first of the punctual concepts, is described as dualistic; the class of objective reals are subdivided into points of space and particles of matter. Here the field of the essential relation consists of the points of space only; these spatial points would be consistent with an absolutistic view of space. The essential relation itself is triadic. Its symbol, R;(abc) is defined to mean "the homogeneous points of absolute space a, b, and c, are in the linear order (or the R-order) abc."[20]

Punctual lines and punctual planes thus become classes of the points of space, as do the figures of geometry. No quantitative distance ideas are introduced, since they can be obtained by definition and the use of projective metrics. A similar treatment of distance is employed in the remaining six concepts.

The axioms of geometry are enunciated in the form given them by Veblen, and are twelve in number. I Hp R[21]

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20. MCMW, 476. Whitehead's statement appears in italics, however, as do all his definitions in the 1905 memoir.
21. To be read "the first hypothesis respecting R." For the complete symbolic statements of the various concepts, see Appendix A at the end of this thesis.

asserts that entities exist which satisfy R;(abc). II Hp R and III Hp R are axioms of order; that the points of space involved in the relation are distinct is assured by IV Hp R. In this manner Whitehead avoided the troublesome difficulty encountered by Kempe and Royce when the two end-points of a segment are identical. In Whitehead's memoir an improvement over the axiomatic statements of Veblen is apparent: Whitehead's axiomatic statement of II Hp R, for instance, when translated from its stated symbolic form, reads, "_If_ _a_, _b_, _and_ _c_ _are_ _each_ _points_, and the point order _abc_ is also given, then order _cba_ is implied."[22] Veblen had considered this possibility, for it had been suggested to him by R. L. Moore, one of the pre-publication readers of "A System of Axioms for Geometry,"[23] but had discarded it. The additional hypothesis, "if _a_, _b_, and _c_ are points," appears in subsequent axioms in Whitehead's, but not Veblen's, treatment.

V Hp R secures the condition that a straight line can be infinitely extended, thereby (in conjunction with the parallel axiom) excluding Riemannian geometry. VI Hp R states the relationships necessary if points on an extended line segment were to be mutually related. VII Hp R

22. MCMW, 478. (Italics not Whitehead's.)
23. Veblen, _op. cit._, 344 note.

establishes the existence of points not on a straight line, making triangles (in the ordinary sense) possible. VIII Hp R is the triangle transversal axiom; IX Hp R secured that Euclidean space is of at lease three dimensions. Space was then limited to three dimensions by X Hp R. Cantor's second axiom of continuity[24] of points on a line is XI Hp R, and the twelfth axiom was the Euclidean parallel axiom in an exceptionally precise form. From these twelve axioms, Veblen had shown the whole of Euclidean geometry deducible,[25] and Whitehead had accepted his treatment as adequate.[26] In the development of all the subsequent concepts, it will be established that the propositions of geometry can be deduced from the axioms of that concept by demonstrating that the twelve axioms of Concept I follow from the axioms of the later concepts.

In Whitehead's two volumes which appeared in close succession following the 1905 memoir, Veblen's axioms and the customary Concept I were presupposed to the exclusion of even a passing notice of the linear concepts.[27]

Having completed the exposition of his axioms, the author launched upon what Victor Lowe has correctly called

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24. Cf. Bertrand Russell, <u>The Principles of Mathematics</u>, 296ff.
25. Oswald Veblen, <u>op. cit.</u>, 343-384.
26. MCMW, 469 note.
27. 1906 <u>The Axioms of Projective Geometry</u>, 1-2. 1907 <u>The Axioms of Descriptive Geometry</u>, 2, 7.

Whitehead's "first criticism of scientific materialism."[28] For, in Concept I, the instants of time did not enter into the field of the essential relation. Neither were the particles of matter included in the essential relation. Consequently, an indefinitely large class of extraneous relations became necessary in order to connect the three classes of ultimate existents: that of triadic relations stating that a particle of matter occupied a point of space at an instant of time. "Thus the classical concept is not only dualistic, but has to admit a class of as many extraneous relations as there are members of the class of particles."[29] In addition to its unwieldiness, such a concept labors under the difficulty that its universe is an unchanging one, whose structure needs to be reappraised at every new instant of time. This problem continued to be one of the central problems throughout the later works of Whitehead, and scientific materialism, alias Concept I, was repeatedly the target of this criticism.

The axioms respecting the essential relation in Concept II are identical with those of Concept I. The concept itself was suggested by Bertrand Russell,[30] and is a simplification of the classical dualistic concept. The

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28. Victor Lowe. 1941, <u>op</u>. <u>cit</u>., 35.
29. MCMW, 480.
30. 1903, <u>op</u>. <u>cit</u>., 468.

simplification occurs in the field of the extraneous relation, which becomes dyadic, holding between a point of space and an instant of time. The particles of matter have completely disappeared, and Concept II becomes monistic. Absolute time and absolute space are again assumed.

"The only relevant function is to establish a many-one relation between all instants of time and some points of space, and the actual material point ceases to be important."[31] The condition of impenetrability is insured by the statement that the logical product of any two relations must have no members. Whitehead immediately countered by asserting that in such a universe it would be necessary for a percipient to sense abstract relationships or this world "would appear to labour under the defect that it can never be perceived."[32]

In the case of Concept II, Whitehead thus implicitly rejected the consequent cosmology on epistemological grounds. The possibility of such a cosmology was revived in 1938 by Sir Arthur Eddington, who asserted, "We observe only <u>relations</u> between physical entities."[33]

The third and fourth concepts are called IIIA and IIIB, inasmuch as the mathematical development of both is

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31. Bertrand Russell. 1903, Ibid., 468.
32. MCMW, 480.
33. The Philosophy of Physical Science, 31.

identical. It is only in the physical interpretations that Concepts IIIA and IIIB diverge. The concepts are Leibnizian in the sense that the points of space are not absolutely located.[34] Thus, the points of space are not members of the field of the essential relation; in their place moving particles of ether are postulated. The instants of time also enter the field of the essential relation, which must consequently be tetradic: R;(abct).[35] It follows that the concept is monistic.

It is conceivable that an alternative dualistic concept introducing particles of matter as another class of objective reals might be postulated. In such a case, it would be necessary to treat the particles of matter as members of the field of an indefinitely large number of extraneous relations. The complexity resulting from locating particles of ether with respect to particles of matter would, however, hamper the usefulness of such a concept. Furthermore, the controversial relation between matter and the ether would arise. Accordingly, not even a passing remark expanding such a concept appeared in the memoir of 1905.

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34. It is interesting to note that at this period, Whitehead's knowledge of Leibniz's writings was not from Leibniz himself, but through the interpretation of Couturat. Whitehead. 1941 "Autobiographical Notes" The Philosophy of Alfred North Whitehead, The Library of Living Philosophers, 3, 10.
35. "The particles of ether, a, b, and c, are in the particle-order abc at the instant t."

It is, however, useful to compare this possibility with the cosmology developed by general relativity. In such a case, not particles of ether, but point-instants would become the ultimate existents for the concept. Furthermore, the time-relation would, of necessity, surrender its autonomy to an enlarged fundamental relation. This new fundamental relation would then order point-instants in a fashion outlined by Einstein. The irreconcilable area of divergence would revolve around the necessity for a non-homogeneous space-time in general relativity, and the assumed homogeneity of Whitehead's moving particles of ether. The concept would be dualistic, inasmuch as there would be a need for material particles. A further discussion of relativistic cosmology would diverge from the purposes of this chapter, so a fuller consideration of Whitehead's theory of relativity will appear in Chapter VI of this thesis. A possible treatment of relativity in terms of "On Mathematical Concepts of the Material World" will also be attempted in the unit on the relativity writings.

The geometrical axioms of Concepts IIIA and IIIB are those of Concept I, except that the essential relation is tetradic; Euclidean geometry necessarily follows. For consistency, Whitehead introduced the additional hypothesis, "if $\underline{t}$ is an instant of time," into the antecedent clauses of the existence axioms, I, VII, and IX Hps R. A thirteenth axiom was added to secure that the particles of ether persist throughout the entire time series, but this had no effect on the geometrical reasoning.

In order to postulate an absolute frame of reference at each instant for the definition of velocity and acceleration, a set of mutually perpindicular "kinetic axes"[36] was constructed. Hence only one tetradic extraneous relation between the mutually perpindicular lines and an instant of time was necessary to describe motion.[37]

With respect to the physical interpretation of Concepts IIIA and IIIB, two alternative developments are possible. Concept IIIA may be defined to be that concept in which the same objective reals (the same particles of ether) continue in the same type of motion. Lord Kelvin's vortex ring theory of matter, where each particle persisted in its Wirbelbewegung is a possible direct application of Concept IIIA. The sponge model of the ether suggested by Professor Hicks, in which each particle of ether per-

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36. A term suggested by W. N. Macaulay. 1897 "Newton's Theory of Kinetics" Bulletin of the American Mathematical Society, 3, 371, and adopted by Whitehead.
37. Whitehead made a further attempt at a memoir which may have been pertinent at this point. It was reported as having been communicated in abstract by Dr. Hobson and was titled, "On the Properties of Hyperspace, in relation to systems of Forces, the Kinematics of Rigid Bodies, and Clifford's Parallels." The only reference the author has seen to this paper is in 1889 Proceedings of the London Mathematical Society, 30, 164. Inquiries to the London Mathematical Society and to Trinity College, Cambridge, show that the memoir is not on file at either place.

sisted in its own rotational motion, is a second application. Similarly, Larmor's knots, characteristic of the particles of the ether, readily lend themselves to an analysis under Concept IIIA.

With Concept IIIB Whitehead suggested a novel interpretation of the universe. Here the persistence of motion lies with a unit of volume, but not necessarily of the same particles of ether within it. Thus each particle of ether does not have a characteristic motion which continues throughout its life-history.

Concept IIIA would have been extremely attractive to theoretical physicists of Whitehead's day, but the memoir attracted no attention.

The difficulty with these two concepts, as with the theories of the ether which were prevalent, is that it is questionable whether apparently solid matter can be defined satisfactorily in terms of rarely constituted ether. Furthermore, they suffered under the presumption of an absolute spatial and temporal frame. On other grounds, Whitehead implicitly rejected both these concepts in favor of the final concepts to be developed.

The remaining three concepts, called linear concepts with respect to the fact that the objective reals have properties associated with a straight line considered as a unit, were all Leibnizian in the same sense as those concepts IIIA and IIIB.

A point of potential criticism applicable to the three "Leibnizian" concepts is that, although a relational theory of space is adopted, an absolute theory of time is utilized. Such criticism would be necessitated with the admission of a four-dimensional space-time manifold. At this time, however, Whitehead was not prepared to offer such a manifold. Furthermore, in order to define motion, it was necessary to introduce the kinetic axes, and these presupposed the existence of an absolute time-series. It may be questioned whether these axes do not also introduce a preferred space-frame as well. Even in his relativity writings, Whitehead was reluctant to abolish absolute time completely. It was primarily on the argument that, for any observer, in some sense an absolute temporal succession was necessary to define motion satisfactorily, Whitehead rejected a completely relational theory of time both here and in later writings.

In the final three concepts, points are derived from classes of linear objective reals. Projective geometry had defined the point as a class of straight lines, but such an analysis Whitehead rejected on the grounds that a descriptive point was implicitly postulated in the projective analysis. There is thus the necessity for the introduction of a new definition of a point in terms of linear objective reals which does not depend on a prior assumption of the existence of that point. Two possible definitions are suggested: the Theory of Interpoints and the Theory of Dimensions. Although H. C. Brown drew attention to the excellence of these two theories,[38] even this section of the memoir attracted no notice. Ten years after the publication of "On Mathematical Concepts of the Material World," Whitehead called the definition of points an "unwritten chapter of mathematics."[39]

It is instructive to note that the problem of the definition of points from other elements without involving a circularity of reasoning was one which was persistently attacked throughout most of Whitehead's later writings. Nowhere else, however, was the analysis explicitly that of the 1905 memoir. The primary reason for this fact, in the opinion of this thesis, is not that Whitehead considered the method of definition offered in this memoir to be intrinsically defective. On the contrary, he referred on several occasions to this memoir as giving one possible method of solving the problem.[40] Rather, the reason is to

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38. 1907 "Review: 'On Mathematical Concepts of the Material World'" Journal of Philosophy, Psychology, and Scientific Method, 4, 50-52.
39. 1915 "Space, Time, and Relativity" Proceedings of the Aristotelian Society, new series, 16, 107.
40. Ibid., 107. 1916 "La Théorie Relationniste de l'Espace" Revue de Métaphysique et de Morale, 23, 435. 1920 "Einstein's Theory: An Alternative Suggestion" The

be found in the fact that the problem was later tackled in terms of "abstractive sets," whose elements are directly concerned with satisfying the demands of epistemology.

The fifth and sixth concepts, named by Whitehead IV A and IV B, proceed under a mathematical development the same for both, and diverge only in the function of the extraneous relations. These two concepts depend for their demonstration on the Theory of Interpoints.

In Concepts IV A and IV B the interpoints[41] are the points themselves. The Theory of Interpoints derives its name from the fact that linear objective reals intersect, but that this intersection can be described without presupposing the descriptive intersection-point. The same linear objective reals do not always intersect each other in the same order in successive instants. Thus, an interpoint is an entity derived from the linear objective reals of which it is the common subclass. This thesis concurs with the opinion of Dr. Victor Lowe that this Theory of Interpoints is the germ of Whitehead's renowned "Method of Extensive Abstraction."[42] The later method will be discussed in Chapter V of this thesis.

(London) Times Educational Supplement, Number 252, 83.

41. "Interpoints" is the abbreviated form of "intersection-points."

42. 1941 op. cit., 39.

In Concepts IV A and IV B the field of the essential relation is the entire class of linear objective reals and all the instants of time. The essential relation itself is pentadic: R;(abcdt).[43] Fourteen axioms are required to establish the possibility of Euclidean geometry in Concepts IV A and IV B.

I Hp R is an axiom expressing the persistence of the linear objective reals throughout the time series; II Hp R establishes the existence of the time-series. The presence of these two axioms is again an indication of the preoccupation of Whitehead in showing a world of change mixed with a world of permanence. III Hp R provides that a linear objective real cannot intersect itself. IV, V, and VI Hps R of these two concepts are directly comparable with II, III, and IV Hps R of Concept I. VII Hp R establishes the membership of any linear objective real in the class of interpoints on that linear objective real, while VIII Hp R establishes an axiom regarding the ordering of interpoints on any objective real. IX Hp R of these two concepts repeats V Hp R of Concept I. The remaining axioms, X through XIV Hps R of Concepts IV A and IV B are directly comparable with VIII Hp R through XII Hp R of Concept I.

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43. Read, "the linear objective real _a_ intersects the linear objective reals, _b_, _c_, and _d_, in the order _bcd_ at the instant _t_."

Concept IV A is the dualistic alternative possible under the preceding development. Instead of a particle of matter, however, Whitehead postulated a "corpuscle" or volume with some special property defining its relationship to the motion of the linear objective reals which may pass through it. A division of the corpuscles into positive and negative entities then becomes a probable physical interpretation. Consequently, an indefinitely large class of triadic extraneous relations are needed to locate a given moving corpuscle with a moving interpoint at an instant of time. In addition, the single tetradic extraneous relation setting up kinetic axes for the purposes of defining velocity and acceleration is necessitated by the moving interpoints. It would be imperative to establish laws of motion for the corpuscles and for the linear objective reals, with the possible condition that these two sets of laws may conceivably interact. Under such a concept it would be possible to dispense with the ether as the conveyer of the lines of force between corpuscles, since the linear objective reals could accomplish such a task. This concept is not greatly superior to the classical dualistic Concept I, but its incorporation of motion into the field of the essential relation between linear entities rendered it closer to the physicists' requirements than any preceding concept.

Concept IV B is monistic, a condition secured by abolishing the material particles associated with a corpuscle. Instead of the triadic extraneous relations of Concept IV A, the extraneous relations of Concept IV B become dyadic, holding between an interpoint and an instant of time. The tetradic relation furnishing the kinetic axes is also necessary. This concept, by abolishing material particles from the class of ultimate existents, invites the same epistemological difficulties incurred in Concept II. Although Whitehead ignored any physical concepts consonant with Concept IV B, it might be noted that Faraday's notion of the lines of force might be readily associated with the linear objective reals.

The seventh of the mathematical concepts of the material world, labelled Concept V, was really the justification for preparing the memoir. That concept depends upon both the Theory of Interpoints and the Theory of Dimensions for its exposition. It has been mentioned that in Concepts IV A and IV B the interpoints were the points themselves. The possibility that the interpoints were only a part, rather than the whole, of a point, leads to an extension of the analysis of the basal concept of a point. As was noticed in Concept IV A, it was necessary in the physical interpretations to subdivide the corpuscles into negatively and positively charged entities. If, then, a point be composed of some entity in addition to the inter-point, such a bifurcation might be rendered less arbitrary. Accordingly, Whitehead investigated the properties of classes of linear objective reals further, and developed his Theory of Dimensions in the hope of obtaining a concept more readily associated with the supposed nature of the physical world.

The Theory of Dimensions thus investigates the interrelationships among classes having certain properties. The development of this theory along with that of Concept V may be an indication of the lines along which the projected fourth volume of Principia Mathematica would have proceeded. Writing of the projected fourth volume at Whitehead's death, Lord Russell reported that

> A good deal of this was done, and I hope still exists. But his increasing interest in philosophy led him to think other work more important. He proposed to treat a space as the field of a single triadic, tetradic, or pentadic relation, a treatment to which, he said, he had been led by reading Veblen.... And generally a space of n dimensions as the field of an (n+1)-adic relation.[44]

The property of the linear objective reals considered by Whitehead is that of "homaloty." Homaloty in this sense is readily comparable with, and is an extension in

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44. 1948 "Whitehead and Principia Mathematica" Mind, 57, 138.

meaning of, geometrical "flatness." "A class of straight lines is flat, when it is a necessary and sufficient condition for membership that a straight line meets two members of the class, not at their point of meeting."[45] It follows then that planes and spaces (as line-loci) are flat. In terms of defined homaloty-subclasses, of "homaloty-primes," and "homaloty-equivalences," the definition of the "homaloty-dimension number" emerged. From this definition of the homaloty-dimension number the condition appears that for Euclidean geometry, the homaloty-dimension number must be three. Thus a geometry of more than three dimensions is excluded from Whitehead's memoir. It must be observed that when properties of linear objective reals other than flatness or homaloty are considered, the dimension number is not necessarily three.

The seventh of the mathematical concepts offered by Whitehead, Concept V, is a monistic Leibnizian[46] concept. For the field of the essential relation it possesses the class of linear objective reals, as well as the instants of time. As with Concepts IV A and IV B, the essential relation is pentadic, and is read as in the two previous concepts: R;(abcdt).

"Homaloty-points" and "homaloty-planes" in Concept V

45. MCMW, 493.
46. Leibnizian in the same sense as Concepts IV A and IV B.

are the products of complex definitions; geometrical points and geometrical punctual planes are special classes "associated with" corresponding classes of linear objective reals. The relation between the homaloty-points and -planes and punctual points and planes rested on the fact that they had certain linear objective reals in common. One or more interpoints are always associated with a homaloty-point, but the converse is by no means true. The part of a point which does not possess an interpoint, the "nonsecant" part, derives its existence from subclasses of lines having no interpoints themselves, but which are members of classes of lines which had interpoints. Hence either interpoints or[47] nonsecant parts might constitute a point.

The axioms of this concept number seventeen due to the dependence of the concept on both the Theory of Interpoints and the Theory of Dimensions.

I Hp R establishes the duration of the linear objective reals throughout the instants of time. II through VI Hps R of Concept V, being interpoint hypotheses, are equivalent to III through VII Hps R of Concepts IV A and IV B. VII and VIII Hps R define the relation of interpoints to points described in the preceding paragraph.

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47. "Or" in the inclusive sense.

IX, X, and XI Hps R establish the homaloty property as a geometrical property. XII Hp R defines the relations between intersecting planes in space.

With XIII Hp R the Euclidean parallel axiom received a new form, apparently never before or since expressed in such a concise manner. Because of the great freedom in the definition of nonsecancy, that notion is not available to define parallelism. Similarly, to use the notion of non-intersection would be to introduce a dangerous ambiguity in the special sense of the word "intersection" used in Interpoint Theory. The term "cogredience" is therefore adopted to express the means whereby universal preservation of the order of points on lines in perspective is effected. This property of cogredience is indeed only a property of parallel lines in Euclidean space. A cogredient point then becomes the common member of the class of linear objective reals cogredient with another linear objective real, including the linear objective real itself. When, and only when, the space is considered to be three-dimensional Euclidean space, these cogredient points are members of the complexly defined homaloty-points. The Euclidean parallel axiom could then assume the simple form: "the cogredient points are homaloty-points."[48] XIV, XV, and XVI Hps R are the axioms treating of the point-order

48. MCMW, 512.

of homaloty-points; XVII Hp R is the axiom of spatial continuity. Ten pages of symbolic demonstration sufficed to show the axioms comparable to those of Concept I.

In addition to the Essential relation and the time-relation, the single "kinetic axes" extraneous relation completed the concept.

Because of the rich range of physical interpretations possible under Concept V, Whitehead devoted several paragraphs to a suggested application consonant with the theory of the lines of force and the then contemporary advances in submolecular research.

It has been noted that in Concept V points partook of a dual nature: an interpoint part and a nonsecant part. Depending upon the combinations present in any point, five primary types of points were possible: (1) no interpoints, a nonsecant part; (2) one interpoint, no nonsecant parts; (3) one interpoint, a nonsecant part; (4) many interpoints, no nonsecant part; (5) many interpoints, a nonsecant part. The case having no interpoints and no nonsecant parts disappeared, because it was not then a point.

Whitehead accordingly suggested that volumes, or corpuscles, containing an excess of interpoints might be defined to be positively charged. Those corpuscles containing an excess of nonsecant parts would accordingly be negative. A simple definition might follow to identify a point of type (1) with a negative electron and a point of type (2) with a "positive electron."[49] Persistence of the existence of either kind of electron then becomes a matter of defining that persistence and relating it to the continuity of motion axiom.

Consonant with the researches of Sir J. J. Thomson, Whitehead suggested, then, a corpuscle-atom consisting of a large positive (interpoint) electron and a finite number of small negative (nonsecant) electrons. The field of force of charged positive or negative electrons then becomes a direct function of the number of linear objective reals shared in common by the point and the positive or negative "electric points."

There is an indirect suggestion that the property of gravitational mass might in some way be correlated with the number of electric points. Then, following hypotheses relating the motion of linear objective reals and the motion of the electric points, a satisfactory theory of gravitation and electromagnetism might follow. The relativity which Whitehead suggested could be easily interpreted as an example of the technique suggested in this paragraph, although Whitehead did not, at the time, mention the likeness of technique.

The objections to Concept V are all of the nature of

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49. MCMW, 524.

the question, "How?" These are precisely the points indicated by Whitehead as needing special attention. Einstein had answered his own "How?" as he proceeded with later refinements, but Whitehead did not return to consider the specific problems raised by his linear objective reals. Indeed, Concept V was empirically imperfect, but it was highly suggestive. Perhaps the largest "How?" that could have been raised in 1906 was that of correlating physical qualities with the properties of the points made up of nonsecant and interpoint subclasses. The question is only intensified when quantum mechanics is taken into account, and raises a question about the fundamental presuppositions as well. Apparently the only response to the challenge is the simple, yet unsatisfying one, "By successive definitional approximations."

There seem to be two primary problems with regard to employing linear objective reals as a key to cosmology: that of their ontological status, and that of their epistemological status. The ontological problem is acute for metaphysicians and asks, "Do the linear objective reals have the importance placed upon them here by Whitehead?" The problem is raised again in the philosophy of organism with regard to Whitehead's emphasis on prehensions.

In the second place, is anything like a linear objective real a component in experience: do they have any perceptual significance? Whitehead nowhere indicates an answer to this question, either positive or negative. Perhaps he would at the time have accepted the later answer with regard to prehensions--that perception arises from prehensions. But there is no reason to suppose that this is the case.

One possible solution suggests itself. In some sense the homaloty-points must be significant for a percipient. The solution must lie in the direction of assuming that because of the participation of the linear objective reals in the homaloty-point, they thereby construct a meaningfulness which does not attach to the linear objective reals themselves in any more than a potential sense. The significance for a percipient must then be transmitted along or by the participating linear objective reals. If and when these once-participating linear objective reals later participate in a complexly defined perceptual point, they are observed by a percipient. Hence perception would in its operation be a process similar to physical interaction. The mind-body problem then reduces itself to be a direct consequence of ontology; this is directly suggested in _Process and Reality_.

The similarities of "On Mathematical Concepts of the Material World" to the 1927 Gifford Lectures, _Process and_

Reality, are so striking as to support an important assertion of this thesis that Process and Reality assumes added meaningfulness when considered as a metaphysical generalization of "On Mathematical Concepts of the Material World." As will be demonstrated throughout the chapters on the Philosophy of Organism, the discussions of those lectures are so similar in nature to the present memoir, that they might be described as the 1905 memoir brought into metaphysics.

Concept V may yet prove useful as a possible unifier of recent physical researches. Several cases immediately suggest themselves as rewarding exposition.

Case I. Assume the validity of the essential relation, the time relation, and the extraneous relation of Concept V. Assume further that they are existentially valid. Identify the various submolecular "elementary particles" with certain types of the five kinds of points postulated by Whitehead. Such an identification would make an electron a dynamic point of type (1), the pure nonsecant point. Define the positron to be a dynamic point of type (2). A proton becomes a moving point of type (5), where there is an excess of interpoints over the appropriate number of nonsecant parts by one. Mesons are defined to be moving points of type (5) carrying an excess of interpoints or nonsecant parts, as the case may demand.

The neutron is a moving point of type (5) wherein there is an equivalence of the appropriate number of interpoints and nonsecant parts. Neutrinos are moving points of type (3), consisting of an interpoint part and a nonsecant part. Appropriate laws respecting the numbers of interpoints and nonsecant parts which may exist in a stable configuration in a point then become necessary. Corpuscles become volumes, with each of which may be associated a point or points of the appropriate type and density of electric charges. With respect to this property of linear objective reals passing through corpuscles, certain qualities responsible for the various types of energy (including mass) need to be postulated. Variations in the configuration of the linear objective reals may become the condition responsible for having one type of energy rather than another. These configurations would be, as the general case, unstable. As the relative positions of the linear objective reals sought a more stable arrangement, certain types of energy would be released. Conversion from one form of energy to another is made possible. This would be equivalent to asserting an interaction between the laws of motion of the linear objective reals and the laws of motion of the various "points" involved.

Case II. Assume the validity of the fundamental relations. Consider the Theory of Interpoints, the Theory of Dimensions, and the geometrical treatment of Concept V as an ideal case which the physical universe tends to approach. Define the basic submolecular entities in a manner similar to that of Case I. The properties of a linear objective real are, however, altered when that linear entity existentially passes through a corpuscle. This alteration may be of the nature of a deflection from the ideally linear locus. There is, then, a resulting distorted configuration of interpoints from the ideal norm dictated by Concept V. The gravitational field is a consequence of this distortion, and persists as the distortion is passed along the linear objective real. Other types of energy are identified by certain structural aberrations, and are convertible in terms of degree and type of variation from the ideal stable configuration of linear objective reals. This case is subject to the same sort of empirical criticism to which a Platonic description of cosmology would be liable. Its verifiability would seem to be impossible, although ideally it would satisfy the demands of predictability.

Further cases also suggest themselves. The exposition of those cases depend, however, on the development of later chapters of the thesis, and will be postponed until the appropriate stages of progress have been reached.

As a cosmological essay, "On Mathematical Concepts of the Material World" had several suggestions of value. In the first place, it gave a first approximation to the problems, methods, and solution of Whitehead's later cosmological works. It is the first expression of faith (subsequent to Universal Algebra) in the power of "systems of thought" as contrasted with piecemeal investigation of particular problems. It also serves, as will be seen in Part Three of this thesis, as a basis for a fuller understanding of Process and Reality. The memoir represents an invective against the classical absolute concept of the material world. Another was suggested by Whitehead himself: it disentangles "the essentials of the idea of a material world from the accidents of one particular concept."[50]

The ontological question of the relation of any of the seven concepts to existence is ignored.[51] Later, in The Axioms of Projective Geometry, Whitehead offered a possible clue in the direction of solving this problem.[52] He denied the efficacy of the ontological argument in deciding the validity of an existence theorem for any concept. Similarly, an inductively produced argument from physical science has no significance for the existence theorem. In arranging such an argument in syllogistic form, there is a glaring undistributed middle term. The alternative proof

50. MCMW, 465.
51. MCMW, 467.
52. Pages 3-4.

suggested by Whitehead is linked with the independence of the axioms in any concept. When a categorical set of axioms has been established, the very proof of their independence would establish a proof of the existence theorem for the appropriate disjunctive set of the axioms. This, however, does not constitute an ontological existence proof.

One important current of thinking demonstrated in this memoir and assumed a central position in all Whitehead's later work was that of asserting the mutual dependence of natural elements. Whitehead's writings may be considered to be a persistent underlining of the belief that the natural universe was essentially intelligible and coherent in its operations. In this particular Whitehead agreed with Arnold, who had also said that natural entities must be mutually interrelated.[53] He disagreed with Arnold's extrapolation denying a unified process to the natural universe.[54]

Because of the logical nature of the 1905 memoir, its contents could be readily assimilated into almost any philosophical system, with the exception of classical materialism. For Bradley, the memoir could represent that "necessary way of happening among our appearances."[55]

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53. 1904 Scientific Fact and Metaphysical Reality, 238, 243.
54. Ibid., 164, 250, 257.
55. Bradley, op. cit., 266.

there was too much of space, time, and matter to allow it to be a description of "Reality."

Fraser might have accepted "On Mathematical Concepts of the Material World" as an explanation of the workings of the material universe and as a secondary indication of the coherent structure of a theistic universe. Indeed, an argument for the existence of God from the order of nature might well have arisen from its fruits.

It seems probable that Royce would have classified the memoir as an example of the fruits of critical rationalism. Royce, it will be remembered, had declared that "pure science has no logical dependence on physics."[56] In Whitehead's paper, however, the implication is that both the laws of physics and the axioms of pure geometry at least, might be derived from some more primitive set of laws. Whitehead set about the task, and his works of the relativity era and the philosophy of organism constitute his answer.

By using appropriate definitions regarding the relation of perception to the structure and behavior of ultimate existents, there would be the possibility of an interactionist epistemology. There is only a superficial case for parallelism.

It has been observed that his treatment of time was a

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56. <u>The World and the Individual</u>, <u>1</u>, 9.

concurrence with Russell in expressing an absolutistic frame of reference for that entity. There is a discrepancy in wanting absolute time and relational space, as will be demonstrated in the following chapters on relativity.

Because of the "instantaneous" nature of time, the process of Bergson would be implicitly rejected in the theory as presented. However, appropriate alterations in the class of ultimate existents and the nature of the time-process would make Concept V useful in a Bergsonian world.

Whitehead had, then, in a memoir which he himself considered one of the best pieces he had ever done,[57] accomplished a theoretical unification of the laws of physics for which Poynting had, six years previously, hoped.[58] Whitehead, in 1905, theoretically reconciled the theories of space and the theories of matter which Einstein accomplished in a different, but far more complete, manner in three successive major stages.

It will become increasingly apparent with the discussion of Whitehead's later works, how important "On Mathematical Concepts of the Material World" was in the development of later Whiteheadian cosmology.

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57. Victor Lowe, op. cit., 34.
58. Poynting, op. cit., 471, col. 2.

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PART TWO

THE RELATIVITY ERA

CHAPTER THREE

THE DEVELOPMENT OF RELATIVISTIC COSMOLOGY

The name symbolizing the unification of physical science is not that of Whitehead, but of Einstein. Born in Ulm in 1879, Albert Einstein had also doubted the sufficiency of Newtonian principles to bear the burden of physics. One of the most glaring inadequacies of Newtonian principles lay in the explanation of experiments designed to investigate the motion of matter (especially the earth) through the postulated ether.

Albert Michelson narrated the results of his Potsdam experiment attempting to find the absolute velocity of the earth through a stationary ether.[1] The expected difference in the times of the journeys of two rays of light respectively parallel and perpindicular to the direction of the motion of the earth was so small as to fall within the limits of observational error. Thus the results of an

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1. 1881 "The relative motion of the Earth and the Luminiferous ether" American Journal of Science, (3), 22, 120-129. It is interesting to note that this experiment was financed by Alexander Graham Bell.

earlier experiment by Fizeau were confirmed, and Michelson suggested that his small positive residuals could be ignored.

Michelson repeated his experiment in collaboration with Edward W. Morley in 1886[2], and again in 1887.[3] On both occasions the results confirmed earlier experiments and violated the only theoretical explanation known. Only very small positive residuals could be found. The same experiment and similar ones, repeated by other investigators at a later date, resulted in the same way.

Sir Oliver Lodge, investigating this paradox, carefully reviewed the theoretical relation between the earth and the ether[4]. As a material body advanced through a stationary ether, it caused a condensation of that ether in front of itself. This ether streamed through the pores of the body with a diminished velocity, and then evaporated to its original state behind the body. Admitting that a Fitzgerald hypothesis that matter contracted in the

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2. 1886 "Influence of Motion of the Medium on the Velocity of Light" The American Journal of Science, (3), 31, 377-386.
3. 1887 "On the Relative Motion of the Earth and the Luminiferous Ether" The American Journal of Science, (3), 34, 333-345.
4. 1894 "Aberration Problems.-- A Discussion concerning the Motion of the Ether near the Earth, and concerning the Connexion between Ether and Gross Matter; with some new Experiments" Philosophical Transactions, (A), 184, 727-804.

direction of its motion was possible, Lodge nevertheless considered the suggestion not proved. Insisting that astronomical aberrations were consistent only with the view that the ether stream must remain unaffected by matter, Lodge asserted the complete independence between ether and matter. However, Michelson's repeated experiments appeared to be consistent only with the theory that the ether near the earth must be relatively stagnant; and no really decisive conclusion was apparent.

The highly respected Dutch physicist of this era, H. A. Lorentz, offered an ad hoc hypothesis by way of explanation of the Michelson-Morley experiment[5], and reiterated it[6] when various other investigators tried different methods of finding the velocity of the earth relative to the ether.

The explanation asserts that (with Fitzgerald), when any particle moves with a velocity v, its length in the direction of motion is contracted by an amount $\beta = c/\sqrt{c^2 - v^2}$, where c is the velocity of light in vacuo. Thus a spherical electron would become an ellipsoid flattened by the amount β in the direction of its motion. The diameter

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5. 1920 "Der Interferenzversuch Michelsons" reprinted in Das Relativitätsprinzip, 1-5. The hypothesis originated, however, with Fitzgerald.
6. 1920 "Elektromagnetische Erscheinungen in einem System, das sich mit beliebiger, die das Lichtes nicht erreichender Geschwindigkeit bewegt" reprinted in Das Relativitätsprinzip, 6-25.

of the earth in the direction of its motion would be correspondingly shortened by two and one-half inches. This hypothesis would indicate the futility of any experiment of an optical nature to detect any traces of the earth's motion through the ether, since those effects would be exactly compensated by the contraction of the matter forming the apparatus. Accordingly, in order to relate two uniformly moving systems, the equations known as the Lorentz transformation were introduced. Consider a coordinate system K' moving with a uniform velocity v along the x-axis with respect to the coordinate system K. Then the values x, y, z, t of system K are related to the comparable values x', y', z', t' of system K' according to the transformations proposed by Lorentz:

$$x' = \beta\,(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \beta\,(t - vx/c^2)$$

In June, 1905, six months before Whitehead presented "On Mathematical Concepts of the Material World" to the Royal Society, Einstein submitted his special relativity theory to the world.[7] It produced a revolution in physical

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7. "Zur Elektrodynamik bewegter Körper" Annalen der Physik, (4), 17, 891-921.

theory which soon gained the support of nearly all the physicists. Two assumptions suggested by experimental evidence sufficed to provide the groundwork of this axiomatic attempt to describe the operation of physical phenomena. (1) Any light-ray moves in a stationary system of coordinates with the same velocity, whether emitted by a stationary or moving source. (2) The Principle of (Special) Relativity asserts that "the laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or to the other of two systems of coordinates in uniform translatory motion."[8] This principle implies that the hitherto fundamental invariants (mass, length, time) no longer had an absolute significance, but were merely an expression of the relation of things in the external world to some specified observer's station. Physical invariants would then be the physical laws, but not the quantities of mass, length, and time. Furthermore, the principle asserts that any recognition of absolute velocity is impossible. Sir Edmund T. Whittaker appropriately named such a principle a "Postulate of Impotence."[9]

But Einstein was by no means the originator of the

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8. Ibid., 895.
9. 1949 From Euclid to Eddington: A Study of Conceptions of the External World, 58. These were the Tarner Lectures for 1947.

idea underlying the principle of relativity. In an address to the International Congress of Arts and Sciences in 1904, Henri Poincaré had emphasized its importance as a fundamental postulate.[10]

It was Minkowski who gave Einstein's theory its final mathematical form, a contribution insufficiently acknowledged by many later writers. Minkowski suggested[11] the welding of space and time into a four-dimensional spatio-temporal continuum; a point of space (x, y, z) at an instant of time (t) would then become a "world-point" (_Weltpunkt_) (x, y, z, t). In order to maintain an equivalence of units, the t-component is multiplied by the critical velocity _c_. Thus, the measure of the separation of two events, called the interval (ds), is expressed by the relation

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2.$$

Since the relative largeness of the last compound term usually causes ds^2 to be negative and less meaningful, the

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10. 1905 "The Principles of Mathematical Physics" _The Monist_, _15_, 5, 9-12. Translated from the French by George Bruce Halsted.
11. 1909 "Raum und Zeit" _Physikalische Zeitschrift_, _10_, 104-111. Cf. G. Burniston Brown: "As early as 1813 Lagrange suggested that mechanics was a four-dimensional geometry, with time as the fourth dimension." 1950 _Science: Its Method and Its Philosophy_, 142.

convention

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

was adopted.

The stream of events in the whole universe, continued Minkowski, thus becomes the description of the world-lines (Weltlinien) generated by the world-points. In this way, he expressed the hope that "physical laws might find their most perfect expression as reciprocal relations between these world-lines."[12] Such a possibility proved so attractive that much of current physical cosmology is an expression of its implications and possible interpretations. The idea has also rooted itself in philosophical cosmology.

A further generalization of Newtonian physics implied in the Special Theory of Relativity was that the apparent mass of a moving body is β times the mass at rest. This assertion provided theoretical confirmation of a finding that the apparent mass of moving electrons is greater than that of stationary ones. Accordingly, Einstein pronounced that if a body radiates energy L, its mass diminishes by the amound L/c^2.[13] Consequently, the mass of a body is nothing more than a specialized form of energy, to be related by the expression

$$e = mc^2.$$

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12. Minkowski, op. cit., 104.
13. 1920 "Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?" reprinted in Das Relativitätsprinzip, 53.

The highly respected principle of the conservation of mass was shown to derive its authority from the more primitive principle of the conservation of energy.

It is well to notice that after this stage of development, some of the adherents of special relativity (notably E. A. Milne) chose a path diverging from that which Einstein followed, and have produced cosmologies which rival Einstein's for supremacy. Indicating this diversity of opinion regarding the next step in relativity theory, Sir Arthur Eddington in 1920 repeated a statement that more than two hundred different theories of gravitation had been offered.[14] Few of these had attracted attention, primarily because no critical experimental verification was suggested.

In several successive stages Einstein approached the formulation of his General Theory of Relativity, in which he examined the phenomena of gravitation in terms of the geometry of the world-lines. As early as 1908[15] Einstein had recognized the generalization necessary for his theory of gravitation, and in 1911 that principle was proposed[16]

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14. Space, Time, and Gravitation: An Outline of the General Theory of Relativity, 64. Eddington's statement is undocumented.
15. Albert Einstein. 1949 "Autobiographisches" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 66. Translated by Paul Arthur Schilpp, op. cit., 67.
16. Albert Einstein. 1911 "Über den Einflusz der Schwerkraft auf die Ausbreitung des Lichtes" Annalen der Physik, (4), 35, 905.

as being responsible for a red-shift in spectral lines and the deflection of light passing through a gravitational field. However, not until 1915 was it held to explain the operation of a gravitational field.

The special theory had shown that the inertial mass of a physical system was a direct function of its energy content. And experimental evidence had shown that inertial mass was, to a high degree of accuracy, equal to gravitational mass. The blending of the two statements led Einstein to the conclusion that the phenomena of gravitation could not be simply explained within the framework of special relativity. The apparent necessity for a suitable gravitational principle was a less restricted axiom, which has received the name of the "Principle of Equivalence." This principle asserts that a gravitational system of limited extension can be considered as equivalent to a system which is accelerated relative to an inertial system.

In expounding the implications of the Principle of Equivalence, Einstein declared that "the general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are co-variant with respect to any substitutions whatever."[17] This

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17. 1916 "Die Grundlage der allgemeinen Relativitätstheorie" Annalen der Physik, (4), 49, 776.

implies that the equations of the laws of physics must apply to systems in any kind of motion: accelerating (including rotating) as well as the special case (hence the name "special theory") of uniform translatory motion.

The generalized expression for the interval (ds) between two events, in space of any curvature and of any number of dimensions can be described as

$$ds^2 = g_{ik} dx_i dx_k,$$

where the indices i and k assume all integral values from 1 to n, and g_{ik} is a symmetrical covariant tensor and a function of the structure of the continuum involved. At the same time g_{ik} replaces the gravitational potential.[18] For four-dimensional space-time, ten distinct g_{ik}'s are possible (since a symmetrical $g_{ik} = g_{ki}$). It is a peculiarity of g_{ik} that it is not independent of the distribution of matter; consequently near matter the four-dimensional continuum deviates from the ideal case of uniform structure. Whitehead considered this a matter of interpretation and violently disagreed. On the Einsteinian interpretation, at a very great distance from matter the uniformity reappears. Therefore, the propositions of Euclidean geometry are not valid in the neighborhood of matter, and

18. As early as 1914 Einstein had recognized the uselessness of a purely scalar gravitational potential.

a non-Euclidean continuum must be adopted. When a particle moves through the four-dimensional space-time, it follows the path of greatest interval-length between the two events, called its geodesic. When the particle moves with the velocity c, the special case where $ds^2 = 0$ arises, and that geodesic is appropriately known as a "null geodesic." It was a consequence of special relativity that only a ray of light can travel with the critical velocity c. Accordingly, rays of light are postulated to travel along the null geodesics.

The continued presence (after transformations) of derivatives of g_{ik} in the interval equation implies that a gravitating (accelerating) field is present. To account for the vanishing of those derivatives would be to account mathematically for the presence of the gravitational field. It is found that when a function of the derivatives of g_{ik}, the mixed Riemann-Christoffel tensor,

$$B^{\rho}_{\mu\nu\sigma} = 0$$

the removal of the gravitational fields has described a completely flat space-time of any number of dimensions. The condition simulates that of an Euclidean world considered to be at an infinite distance from all matter and all forms of energy.

By contracting $B^{\rho}_{\mu\nu\sigma}$ the condition

$$G_{\mu\nu} = 0$$

appears, which holds in non-Euclidean, as well as Euclidean space. By contrast to the world of $B^{\rho}_{\mu\nu\sigma} = 0$, this world is one remote from, but in the neighborhood of, matter, light, or electromagnetic fields.

Since, even in general relativity, the effects found of the gravitational field are due to the presence of ponderable matter, a thorough consideration of the law of the gravitational field entails an accounting for this effect. Einstein suggested the equation

$$G_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} G = -\kappa T_{\mu\nu}$$

as best qualified to describe the law. The additional tensor $T_{\mu\nu}$ accounts for the energy density of the electromagnetic field and of ponderable matter, while κ is a constant related to the Newtonian gravitational constant.

One of the interesting consequences of the general theory is that gravitation occupies a pre-eminent place among natural phenomena in that the ten g_{ik}'s defined the metrical properties of a field at the same time that they were responsible for the gravitational properties of that field.

The new gravitational law was easily interpreted in the light of Hilbert's principle: "all physical happenings in the Universe are determined by a scalar world-function $\mathfrak{H}$, being, in fact, such as to annul the variation of the integral $\iiiint \mathfrak{H}\, dx_0 dx_1 dx_2 dx_3$."[19] The interpretation of the general theory that gravitation represents the attempt of the universe to straighten itself out was a natural one.

The most significant prediction of the theory arose from its description of the laws of planetary motion. Newtonian physics described, with reasonable accuracy, the motion of all the planets except that of Mercury. The observed advance of Mercury's perihelion was 574" per century; after accounting for the perturbations from all possible sources, a residual advance of 42.9" per century remained unexplained. Many popular commentators on relativity have mistakenly assumed the 42.9" to be the entire advance of Mercury's perihelion.

Leverrier, who had predicted the appearance of the planet Neptune from the residuals in the motion of Uranus, postulated a planet interior to Mercury. Later this expected planet had been christened Vulcan. Einstein's theory, however, accounted for the 42.9" advance without the necessity for postulating an undiscovered planet.[20] Corrections for other planets are smaller:

Venus	8.6"	per century
Earth	3.8"	per century
Mars	1.35"	per century.

19. Sir Edmund T. Whittaker. 1927 "The Outstanding Problems of Relativity: Presidential Address to the British Association for the Advancement of Science, Section A."
20. Einstein, op. cit., 804, 822.

The case of Venus falls within the limits of observation, and Sir Arthur Eddington suggested that it may represent a real discordance within the theory.[21] The case of Mars represented a slight improvement, however.[22] The additional amount of advance contributed due to the effect of solar axial rotation is of little consequence: in the case of Mercury it amounts to only 2.62×10^{-4} times the amount of the advance of the perihelion.[23]

A second consequence of importance arising from general relativity is its prediction of an observable effect of the action of a gravitational field on a ray of light passing through that field. Because of the deflected path of the geodesics around a gravitating body, a ray of light from an infinitely distant source would be deflected toward the center of the gravitating body by an amount proportional to the strength of the field. This would cause the light-source to appear to be displaced in an outward direction from the gravitating body. In the case of a ray of light passing the edge of the planet Jupiter, such a deflection would amount to 0.017", far below the range of

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21. Arthur S. Eddington. 1923 _The Mathematical Theory of Relativity_, 90.
22. Arthur S. Eddington. 1918 _Report on the Relativity Theory of Gravitation_, 52.
23. W. de Sitter. 1916 "On Einstein's Theory of Gravitation, and its Astronomical Consequences, I" _Monthly Notices of the Royal Astronomical Society_, _76_, 727.

experimental verification.[24] In the case of a ray of light grazing the sun, however, Einstein predicted a deflection of 1.74", a value entirely within the range of observation.[25] The simple Newtonian deflection was half this value, and the divergence between the two predictions promised another opportunity for confirmation of the general theory. Fortunately, the opportunity of a total eclipse offered itself on May 29, 1919, and two photographic expeditions set out, one to Sobral in North Brazil and the other to the Isle of Principe near the Cameroons coast. The African plates gave fairly good confirmation,[26] averaging 1.98" ± 0.12". However, the first results from Brazil cast their weight greatly in favor of the Newtonian estimate. But seven plates from Brazil, when measured, gave an average value of 1.61" ± 0.30". The evidence was good, but not conclusive. Accordingly, the observations of an eclipse at the Lick Observatory in 1922 promised to be decisive. The results gave excellent confirmation of the predicted values. The Malayan eclipse of May 1929, however, yielded results ranging from 2.0" to 2.2". The Australian results were re-examined, and were found, upon

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24. Arthur S. Eddington. 1917 "Einstein's Theory of Gravitation" _Monthly Notices of the Royal Astronomical Society_, _77_, 380.
25. Einstein, _op. cit._, 821.
26. Arthur S. Eddington. 1920 _Space, Time, and Gravitation: An Outline of the General Theory of Relativity_, 118.

calculation, to read 2.2". Although the results are closer to the general theory's prediction than to the Newtonian amount, they cannot be construed as giving definite experimental confirmation of Einstein's theory. Professor E. Finlay-Freundlich has also expressed this doubt.[27]

One unfortunate difficulty in effecting a thoroughly satisfactory observation on this critical point is that data on stars whose distance from the sun exceed 7.5 solar radii are almost valueless.[28] And the case is that the number of stars falling inside this region is very small, and they are not symmetrically arranged.

The third phenomenon which might confirm general relativity was proposed by Einstein in 1911.[29] On the basis of the Equivalence Principle (here introduced for the first time), he predicted that spectral lines of sunlight, as compared with corresponding lines from terrestrial light sources, would be displaced toward the red end of the spectrum by the amount $2.12 \times 10^{-6} \lambda$. In 1917 St. John examined the spectral lines of cyanogen and found them not

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27. 1950 August 28. Letter to the author.
28. E. Finlay-Freundlich and W. Ledermann. 1944 "The Problem of an Accurate Determination of the Relativistic Light Deflection" Monthly Notices of the Royal Astronomical Society, 104, 44.
29. "Über den Einflusz der Schwerkraft auf die Ausbreitung des Lichtes" Annalen der Physik, (4), 35, 905.

appreciably deflected[30]; some were even displaced toward the violet. Calcium vapor also gave disappointing results, and observations on iron gave but one-third the predicted value.[31] A further very slight reddening of spectral lines on the sun's limb with respect to its center was expected (the limb effect), and appeared. However, other causes than the relativity effect may have been responsible.[32]

Commenting on the present stage of the investigation of the red-shift of the spectral lines of the sun, Professor Finlay-Freundlich observed,

> ...up till now, it has not been possible to prove convincingly that the solar lines show a general shift...as predicted by the theory of relativity.
>
> It is even very unlikely that this possibility of a verification of the theory will be successful at all in the future, since there are various competing effects which are difficult to disentangle.[33]

And, with respect to a similar effect which should be in evidence in stellar lines, he further warned:

> The statistically derived general red-shift of the B-Stars, called the K-term which more than 30 years ago I had interpreted as a gravitational effect also does not give an unambiguous proof in favour of the theory of relativity, because also this effect has turned out to be of a very complex nature; it is, for instance, strongly

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30. Charles E. St. John. 1917 "The Principle of Generalized Relativity and the Displacement of the Fraunhofer Lines toward the Red" The Astrophysical Journal, 46, 254.
31. Ibid., 261.
32. Ibid., 265.
33. 1950 August 28. Letter to the author.

indicative of the rotation of the galaxy, respectively of the orbital motions of the stars in the galaxy.

The much quoted red shift of the lines in the spectrum of Sirius's companion is quantitatively extremely inaccurate and thus not available for a quantitative test of the theory.

To sum up, it is true that all observations of the sun and stars seem to indicate the existence of an effect as predicted by the theory of relativity, but to separate this effect clearly from other overlapping shifts appears at present rather hopeless.[34]

Other less critical effects are implied by the theory. In 1920 the Russian Fokker found that general relativity implied the addition of a very small amount of time to that required in the precession of the equinoxes.

A further correction to astronomical calculations arose with respect to the advance of the perigee of the moon. The value predicted from general relativity is 1.94" per century, an amount below the observational limit. With respect to possible confirmation of this prediction, Eddington warned, "There are well-known irregular fluctuations in the moon's longitude which attain rather large values, but it is generally considered that these are not of a type which can be explained by any amendment of gravitational theory."[35]

In his Presidential address to the British Association

34. 1950 August 28. Letter to the author.
35. 1923 The Mathematical Theory of Relativity, 99.

in 1927, Sir Edmund T. Whittaker summarized three additional and less obvious consequences of Einstein's theory. The deflection of light rays around gravitating bodies may be so pronounced in certain cases that the light ray will be permanently held by the attracting body, which it would approach spirally and asymptotically.

Again, if there be any electrified particle at rest in a varying gravitational field, that particle must emit radiation.

A third consequence of the theory must be that every particle must have an impermeable ring-fence surrounding it. Writing on this aspect of the problem in 1949, J. L. Synge used a theoretical test-particle to bomb another particle.[36] He found that, after suitable transformations, the test particle would reach the velocity of light as it reaches the center of the bombarded particle. Hence, with $r=0$ as the center of the particle, the "passage of a test particle through the singularity $r=0$ may be uniquely defined."[37] Thus, even in general relativity, the ring-fence can be penetrated.

Immediately following the publication of the general

36. 1950 "The Gravitational Field of a Particle" Proceedings of the Royal Irish Academy, (A), 53, 83-114.
37. Ibid., 108.

relativity theory, two large-scale movements in theoretical physics gathered momentum. The first, the construction of a cosmological scheme based on general relativity, flowered quickly. The second, the search for a unified field theory to account for the phenomena of electromagnetism as well as those of gravitation, proceeded more slowly under the ominous shadow of quantum mechanics.

The Cosmological Models

Investigating his original laws of the gravitational field,

$$G_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}G=-\kappa T_{\mu\nu}$$

Einstein objected that these equations still possessed a solution when the universe was considered devoid of ponderable matter. In order to remedy the situation he introduced[38] a term containing what became known as the "cosmological constant." The introduction of this arbitrary constant, λ, which depends upon the existence of a "world-matter" filling all space, gave the equations the new form:

$$G_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}(G-2\lambda)=-\kappa T_{\mu\nu}$$

The new addition to the theory implied that the

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38. 1917 "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie" Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften (Berlin), 151.

world-matter, when considered on a large scale, would be uniformly distributed in a spherical space.[39] Eddington has urged the objection that there is the theoretical necessity for assuming far more matter to exist than astronomical observations justify.[40] That space is spherical in its nature implies that a ray of light passing along the outer boundaries of the universe will eventually return to its point of origin. Furthermore, as de Sitter explained,[41] ideally the image of the opposite side of the sun at an earlier time should be visible at the point antipodal to the sun's position. That such a phenomenon is not discoverable may, however, be due to the absorption of the light rays along the journey. When generated throughout an infinite stretch of time, the universe might be represented as a cylindrical world in four-dimensional space-time. Because of the variable curvature of space-time, such a cylindrical world would represent, in its sphericity, only a rough approximation. Considered, therefore, in its spatial extensions, the universe would be of finite extent. In its temporal extension, howéver, it would be endless;

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39. Ibid., 149.
40. 1920 Space, Time, and Gravitation: An Outline of the General Theory of Relativity, 162.
41. 1917 "On Einstein's Theory of Gravitation, and its Astronomical Consequences: III" Monthly Notices of the Royal Astronomical Society, 78, 3-28.

there would be no uniquely defined moment of creation or termination of the cosmos. In a sense, therefore, a separation of time from space has been re-introduced. The passage of a particle through this spatio-temporal continuum would be defined by its world-line, and would produce a groove-like track in its route. This world-model further carries the implication that once the distribution and the laws of motion for each particle are given, the entire past and future of the world-model are absolutely determinable. It must be remembered, however, that this world-model is arbitrarily restricted by excluding electromagnetic fields.

Silberstein rejected Einstein's abandonment of the original infinitely large homaloidal world,[42] and further insisted that Einstein's new cylindrical world led to the conclusion that every homogeneous body can have only a spherical shape.[43]

The Dutch astronomer, de Sitter, was most outspoken against the cylindrical world. In a memoir coming soon after Einstein's innovation, he demonstrated a solution of the new equations which would hold even though no matter

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42. 1918 "Planetary Motion in Space-Time of any Constant Curvature according to the generalised Principle of Relativity" Monthly Notices of the Royal Astronomical Society, 78, 366.
43. 1918 "Bizarre Conclusions derived from Einstein's Gravitation Theory" Monthly Notices of the Royal Astronomical Society, 78, 466.

existed.[44] The necessity for the cosmological constant was removed, but the new matterless de Sitter world was nevertheless retained as a cosmological model.

The de Sitter world can be represented as an empty four-dimensional continuum of space and imaginary time forming a spherical surface. When real time, however, is adopted, sections of the continuum form a four-dimensional hyperboloid. Both this and the Einstein cylindrical world are steady-state cosmological models.

Followers of Mach, however, rebelled at the thought of a world without matter, and totally rejected the empirical possibility of the de Sitter world; apparently this is the reason for Einstein's refusal to accept the solution.

A further interesting cosmological model is one in which $\lambda < 0$, and the world-model is static and spatially homogeneous. Such a model has been suggested by Kurt Gödel,[45] and would be known as a rotating universe. One of the amazing properties of such a universe would be that it would be theoretically possible to relive past experiences and to affect those experiences so that they would

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44. 1917 "On Einstein's Theory of Gravitation, and its Astronomical Consequences: III" Monthly Notices of the Royal Astronomical Society, 77, 3-28.
45. 1949 "A Remark About the Relationship Between Relativity Theory and Idealistic Philosophy" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 560-561.

happen in a way different to those which had previously occurred. A red-shift of distant nebulae would not appear unless the rotating universe were also expanding. Empirically, this theory seems impossible; it is, however, an interesting case.

In 1939 G. C. McVittie resumed the defense of the hyperbolic universe of infinite extent. The only postulate which will dismiss a hyperbolic universe in general relativity, he affirmed, would be that there are large quantities of unobservable matter in the universe.[46] Such a postulate would mean the introduction of an experimentally unverifiable _ad hoc_ hypothesis--always an unwelcome stranger.

Meantime, Sir Arthur Eddington, who had been responsible for a considerable part of the relativity literature, had begun to investigate the more philosophical problems rooted in relativity. Possibly under an influence of Whitehead,[47] Eddington emphasized the fact that the notion of a point-event is the fundamental concept in relativity. He diverged from the Whiteheadian thesis, however, in

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46. "Observation and Theory in Cosmology" _Proceedings of the Physical Society of London_, _51_, 536.
47. Whitehead's contribution to the symposium, "Time, Space and Material, Are they, and if So, in What Sense, the Ultimate Data of Science," in which he emphasized the primacy of the event, was published in the _Aristotelian Society, Supplementary Volume II: Problems of Science and Philosophy_, in 1919. Whitehead's more comprehensive _An Enquiry Concerning the Principles of Natural Knowledge_, appeared the same year. Eddington's contribution is dated 1920 (see footnote 48).

asserting that it was an indefinable concept and beyond the range of human understanding.[48]

Because of the close interrelation of matter and space-time in general relativity, Eddington suggested that the presence of "mind" in the universe may be more fundamental than was generally supposed.

> I am almost inclined to attribute the whole responsibility for the laws of mechanics and gravitation to the mind, and deny the external world any share in them. So far as I can see, all that Nature was required to furnish is a four-dimensional aggregate of point-events; it should be possible to pick out a set of entities which would serve as point-events, however badly Nature had managed things in the external world. For the use made of point-events the mind alone is responsible.[49]

By 1938, Eddington had developed this concept to the philosophy of "Selective Subjectivism," based upon his interpretation of relativity that physical hypotheses had to some extent been replaced by epistemological principles. Using these principles, Eddington presented an overwhelming case for his fundamental thesis. The basic equations of physics, in which quantum mechanics plays a large part, he formed from four dimensionless natural constants, one of which was his famous cosmical number--the number of elementary particles in the universe. These constants he

48. 1920 "The Meaning of Matter and the Laws of Nature According to the Theory of Relativity" Mind, new series, 29, 147.
49. Ibid., 155.

believed to be capable of unambiguous a priori prediction.[50] His reconstruction of ordinary physical constants on the basis of these natural constants gave surprisingly close approximations to their observed values.

Eddington's scheme rests on the assumption that the various species of elementary particles investigated by physical science are only subspecies of a set of essentially similar fundamental particles. This assumption is not completely justified on the physical level, but it is nevertheless an assumption which Whitehead also chose to defend in his philosophy of organism. But in comprehensiveness, Eddington's cosmological system rivals any that physical science has been able to derive. It also has the signal advantage of drawing equally from relativity theory and quantum mechanics, an advantage not to be lightly dismissed.

Meantime Hubble had found that the other galaxies are receding from that one in which the solar system is located with a velocity roughly proportional to their distance from it. The immediate conclusion is that the universe must be not of a static nature, but expanding. Lemaître showed the cylindrical world to be unstable: upon the introduction of a disturbing force, the Einstein world would begin to contract or to expand. Lemaître showed that upon

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50. The Philosophy of Physical Science, 58.

expanding, the Einstein universe would tend to distribute its material content, approaching a zero density as a limiting case. It is, of course, possible that the Einstein universe may begin to contract after reaching a limit of expansion.

The question of the curvature of cosmological models was also drawn into question. Although Einstein had been convinced that the flat space-time of special relativity could not form a suitable stage for the operation of gravitation, others (including Whitehead) were less doubtful. Many were dissatisfied with the non-homogeneity of space-time in general relativity; others believed that the accounting for the conservation of angular momentum was accomplished by an unnatural means.

With respect to using flat space-time, it was established that if the tensor g_{ik} is only the gravitational potential, and if an additional tensor γ_{ik}, describing the metric of flat space were supplied, then Einstein's general relativity could be validly interpreted in flat space-time. The improvements in the resulting theory are primarily formal. A more straightforward formulation of the laws of conservation of energy and momentum is possible. In general relativity of flat space-time, the tensor g_{ik}, unhampered by the metric tensor γ_{ik}, produces the alteration of the local velocity of light rays. This distinction is one which is entirely absent from general relativity. Because of the presence of two contrasting tensors, there may be the further possibility of supplying an explanation of the small positive residuals in the Michelson-Morley experiment.

Dirac, rejecting positive and negative curvatures of space as undesirable, adopted flat space in 1938.[51] A basic assumption was that the dimensionless constants of nature discussed earlier are not fixed, but vary roughly in proportion to the age of the epoch.

A. Papapetrou, in adopting flat space-time in 1947, was concerned with general relativity's treatment of the conservation of angular momentum.[52] He declared that only by restricting the g_{ik} and introducing the flat-space metrical tensor γ_{ik}, could angular momentum be satisfactorily explained.

Professor George Temple in 1925 presented a paper[53] which he claimed could be responsible for all the predictions usually regarded as crucial tests of special and general relativity. He assumed the independence of flat space and time, in violent contrast to orthodox relativists.

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51. P. A. M. Dirac. 1938 "A new basis for cosmology" Proceedings of the Royal Society of London, A, 165, 205.
52. 1948 "Einstein's Theory of Gravitation and Flat Space" Proceedings of the Royal Irish Academy, A, 52, 11-23.
53. "On Mass and Energy" Proceedings of the Physical Society of London, 37, 269-281.

Three further postulates were introduced: (1) Newtonian dynamics is valid, again a violent contrast, (2) Maxwellian electrodynamics is valid, and (3) the crucial and most important postulate that "variations in the potential energy of a body...are always accompanied by proportionate changes in its mass."[54] The hypotheses were of such a nature as to be attractive to the experimental physicist, but the memoir attracted little attention. The additional advance of 42.9" per century to the perihelion of Mercury followed from the Temple postulates; so did the deflection of a ray of light in a gravitational field. The shift of the Fraunhöfer lines, however, was calculated to be twice the amount predicted by Einstein, thus representing a more pronounced divergence from observation than the overestimate of general relativity. A point of improvement, however, lay in the production of formulas coinciding with those arising from the Bohr-Sommerfeld theory of the fine-structure of hydrogen and ionized helium, and with those describing the action of electrons in an electromagnetic field. Eddington, who heard the reading of the memoir, suggested the most likely criticism of the theory: that it represented the addition of arbitrary material to classical theory.[55] Nevertheless, it signalled an appreciation

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54. Ibid., 269.
55. 1925 Proceedings of the Physical Society of London, 37, 281.

of Poynting's ideal of describing the sensible in terms of the sensible.

"It has become clear," Sir Edmund Whittaker pronounced, "that General Relativity does not stand alone but is a member of a family of theories that have many features in common. In this family the most serious rival to Einstein's theory is E. A. Milne's Theory of Kinematic Relativity."[56]

Kinematic relativity was first given a full exposition by E. A. Milne in his 1933 Aberystwyth Lectures.[57] Insisting that the current relativistic cosmologies implied basic conditions which were contrary to observational evidence, Milne suggested a totally new beginning. Existing relativities implied that matter must be annihilated or destroyed within the experience of any observer. A modification of the laws of geometry with the curved space-time solutions implies a corresponding modification of the laws of the gravitational field and vice versa. A different geometry exists for each possible gravitational system. The fusion of space and time obscures the undeniable fact of the passage of time. It is necessary to assume as a physical fact the constancy of the velocity of light _in_

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56. 1950 Book Review of _Albert Einstein: Philosopher-Scientist_, _Scientific American_, _182_, 57.
57. Published in 1935 under the title, _Relativity, Gravitation, and World-Structure_.

vacuo. The universe must consist of a finite number of particles. All these conditions, which were representative of the necessary conclusions of orthodox relativistic cosmology, Milne held to be either artificial distinctions or directly contrary to experience.

Abandoning the notion of the homogeneous distribution of matter in the universe on the grounds that motion would thereby be rendered impossible, Milne adopted as his "Cosmological Principle" a postulate to replace homogeneity. The cosmological principle asserts that two observers on equivalent particles in a given system will give coinciding descriptions of that system. A comparison with Hubble's "sample principle" suggests itself. The sample principle asserts that two equivalent portions of the system will exhibit the same general characters. The extension of the sample principle to a description by equivalent observers of the *entire* system can then be identified with the cosmological principle. Consonant with this, Milne constructed a simple kinematic world-model adopting a flat space, in which all the particles in the universe were already in existence.[58] Although accepting the theory that galaxies are receding with a velocity proportional to their distance from the observer, he charged that the interpretation that space itself is expanding is meaningless. It

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58. E. A. Milne. 1935 *Relativity, Gravitation, and World-Structure*, 9.

is simply a question of describing the actual motions of astronomical bodies.

In kinematic relativity, two time-scales are of importance: t-time and τ-time. The clarification of this distinction is due to Milne. Time expressed in terms of τ is the Newtonian time-scale, used for simplicity in physics; the age of the universe and its volume, according to an ordinary τ-clock, are infinite. The second time-scale, however, is that marked out by atomic processes, e. g., radioactivity. The relation between the two times may be expressed by the equation,

$$\tau = t_0 \log (t/t_0) + t_0,$$

where t_0 is the present age of the universe on the t-scale, about 2×10^9 years.[59] It is in terms of t-time that Kinematic relativity displays its most interesting features.

Around any given observer the universe will always appear spherically symmetrical; every particle is surrounded by other particles. The particle-density around each particle will be locally homogeneous,[60] but will tend to increase as the radius of the included area increases. The universe itself will have a radius $r = ct$, where t is

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59. E. A. Milne. 1944 "On the Nature of Universal Gravitation" Monthly Notices of the Royal Astronomical Society, 104, 134.
60. A condition whose validity is questioned by Shapley.

the age of the universe in the appropriate time-scale.[61] Furthermore, at this outer boundary, the particles will be advancing radially outwards with the speed of light. Because of this critical speed at the boundaries, they will never be reached by any particle not travelling with that speed. At the outer boundaries of the universe ($r = ct$) the density will tend to infinity. Consequently, the total number of particles, their total mass and energy, will all be infinite, although the volume is of finite, but expanding size. It is a consequence of this condition that the laws of gravitation will be independent of Einstein's cosmological constant. By virtue of its infinite number of particles, it presents a contrast to Eddington's expanding universe of a constant finite number of particles.

It is because the quanta of light emitted earlier will have a decreased frequency that the red-shift of the solar or stellar spectra might be expected.[62] Thus, an empirical result predicted by Einstein as a result of one condition appears also in kinematic relativity, but with a wholly different justification.

The density at any place in the universe will diminish as a function of time, since every particle is in uniform

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61. 2×10^9 years.
62. E. A. Milne. 1944 "On the Nature of Universal Gravitation" Monthly Notices of the Royal Astronomical Society, 104, 134.

outward motion. There is, however, no ultimate state of the universe as a whole, although the system is, in a sense, deterministic.

Considering the condition $t=0$, Milne found that his system, in common with the other expanding models, would reduce to a point. However, he explicitly declined to accept Lemaître's hypothesis of a super-radioactive atom at $t=0$.[63] The origin of the contents of the universe may possibly be attributed to a First Cause. But, he warned, "once we have added a First Cause, our system sets no limit to the further activities of this First Cause, for we have left room for unending experiments in evolution."[64]

Einstein's rejection of his strongest rival relativity theory is based upon disagreement with the scope of Milne's fundamental principles. Specifically directing comment upon kinematic relativity, Einstein wrote,

> Concerning Milne's ingenious reflections I can only say that I find their theoretical basis too narrow. From my point of view one cannot arrive, by way of theory, at any at least somewhat reliable results in the field of cosmology, if one makes no use of the principle of general relativity.[65]

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63. Ibid., 340.
64. Ibid., 139.
65. 1949 "Remarks Concerning the Essays Brought Together in this Co-operative Volume" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 684. Translated by Paul Arthur Schilpp.

W. H. McCrea, also adopting flat space and time, has defended the steady-state theories particularly with respect to the creation of new matter in terms of the spontaneous emergence of hydrogen atoms. In McCrea's steady-state solution, he extended the cosmological principle to include the time-dimension, arriving at the "perfect cosmological principle."[66] Accordingly, a constant density of world-matter can be accomplished by balancing the creation of new matter with a recession constant describing the expansion of the universe.

Summarizing the importance of relativity, Einstein has written,

> The eminent heuristic significance of the general principle of relativity lies in the fact that it leads us to the search for those systems of equations which are in their general covariant formulation the simplest ones possible; among these we shall have to look for the field equations of physical space.[67]

The Unified Field Theories

It was noted earlier (pages 120-121 of this thesis) that two problems attracted the physicists after the publication of general relativity. The construction of

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66. 1950 "The steady-state theory of the expanding universe" Endeavour, 9, 3-10.
67. 1949 "Autobiographisches" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 68. Translated by Paul Arthur Schilpp, op. cit., 69.

cosmological models has been considered; the problem of constructing a unified field theory therefore commands attention.

That a successful unified field theory which will provide critical tests is seriously doubted by many physicists. Even severer doubts arise on the question of the possibility of reconciling relativity and quantum mechanics as they now stand, but this second and extremely troublesome problem will be postponed until the end of the chapter. But the justification for attempting a unification lies in the fact that without unification, the gravitational field is essentially conceived (by the orthodox relativists) as a geometrical field, and the electromagnetic field as a physical field. The bifurcation is undesirable. Unified field theories attempt to describe the electromagnetic field as a geometrical field as well. Whitehead would defend the attempt to unify on the basis of making them both physical fields.

In order to prepare for the erection of a unified field theory, certain considerations involving the geometrical continuum which were of no concern in general relativity, must be satisfied. The geometry of general relativity, as well as general relativity itself, is strictly valid only in the absence of an electromagnetic field. When the geometrical continuum is further generalized, a much broader conception of parallelism is possible, as was demonstrated in 1917 by Levi-Civita. In order to calculate the interactions of a physical field, it is necessary to describe the possible translations of the representative vectors. Such a requirement can be accomplished by means of a "parallel transport," whereby the properties of the field are unaltered. By means of a generalized definition of parallel transport in the affine geometries, a family of curves called loxodromes can be considered to be responsible for describing the action of the electromagnetic field.

One of the earliest attempts to form a unified field theory was that of Edwin B. Wilson and Gilbert N. Lewis, published in 1913,[68] and which Whitehead acknowledged to be a stimulus to his own relativity writings.[69] Using a four-dimensional non-Euclidean geometry of space-time, the authors demonstrated the laws of electromagnetism and gravitation to be capable of simple interpretation in terms of this geometry. Congruence was defined in terms of parallel vectors, but the notion of a parallel transport was not yet presented. A novel aspect of the theory

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68. "The Space-Time Manifold of Relativity. The Non-Euclidean Geometry of Mechanics and Electromagnetics" Proceedings of the American Academy of Arts and Sciences, 48, 389-507.
69. 1919 An Enquiry Concerning the Principles of Natural Knowledge, vii.

was the introduction of a four-dimensional vector, called the "vector of extended momentum." This new vector was defined to be the product of the unit tangent[70] to the world-line[71] of a material particle and the mass[72] of the particle. By asserting the conservation of extended momentum, the conservation laws for momentum, mass, and energy follow. The fields arising from the vector of extended momentum can be identified with the fields of gravitational operation.

Soon after general relativity was published, Ernst Reichenbächer submitted an attempt at unification, but found a difficulty in finding a natural division of entities which might account for a divergence of fundamental electrical particles.[73]

But the first attempt at unification to attract considerable attention was that of Hermann Weyl, who adopted the generalized geometry of Levi-Civita. From the geometrical structure Weyl showed that the Maxwellian equations followed naturally,[74] but in place of Einstein's simple

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70. A vector quantity.
71. Called by Wilson and Lewis a (δ)-curve.
72. The scalar quantity determined by a person at rest relative to the particle.
73. 1917 "Grundzüge zu einer Theorie der Elektrizität und der Gravitation" Annalen der Physik, (4), 52, 134-173.
74. 1918 "Gravitation und Elektrizität" Sitzungsberichte der Preussischen Akademie der Wissenschaften (Berlin), 465-480.

gravitational equations, a more complex form emerged. The difficulty of reconciling the existence of various kinds of elementary particles with the geometrical demands was still unsolved. In his definitive discussion on the relativity theories arising from a series of lectures in Zürich in 1917, Wely suggested that in his own newly introduced "relativity of gauge" the fact that elementary particles have the same quantity of charge and mass is a consequence of the fact that they are "embedded" in a common world.[75]

But the decision of later writings is that the standing difficulty in the way of adoption of Weyl's, as of any other, unified field theory, is that it fails to designate some consequences in addition to the Maxwellian equations which may be tested for validity. Although they accomplish their task of providing a unification of the laws of gravitation and electromagnetism, they fail to exceed the grounds already opened by the simpler general relativity and Maxwellian electrodynamics.

Four years later Weyl appended a note to republications of his "Gravitation und Elektrizität" the admission, "I do not believe that the problem of matter is to be solved by a mere field theory."[76]

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75. Space-Time-Material, translated by Henry L. Brose, 300.
76. "Gravitation and Electricity" The Principle of Relativity, 216.

Weyl's work had attracted Eddington's acceptance, and in 1920 the latter scientist considered the marriage of gravitation and electricity to have been effected.[77]

An even more generalized infinitesimal geometry was developed by Wilhelm Wirtinger, who disclaimed the need for the assumption of a metric geometry.[78] One primary difficulty arising from such a geometry lies in its point of application to physical observation. It may, however, be a powerful method of analysis.

One of the most recent unified field theories is that of J. Podolanski,[79] in which the fundamental task of electromagnetism is accomplished by raising the number of dimensions to six. By restricting the connections of the six-dimensional manifold through the agency of a "structure axiom" based on the parallel transport notion, he decomposed the continuum into a four-dimensional family of flat laminations. The interpretation therefore follows that the world of "events" takes place in the four-dimensional spaces normal to the laminations. Because of this

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77. 1920 Space, Time, and Gravitation: An Outline of the General Theory of Relativity, 174.
78. 1922 "On a General Infinitesimal Geometry, in Reference to the Theory of Relativity" Transactions of the Cambridge Philosophical Society, 22, 440.
79. 1950 "Unified field theory in six dimensions" Proceedings of the Royal Society of London, A, 201, 234-260.

four-dimensional immersion in the six-dimensional continuum, two fields of inertial forces arise. One is defined to be responsible for the Maxwellian field, the other possesses negative energy capable of nullifying the singularities of the first.

The use of a spatio-temporal continuum of dimensions higher than four has been popular, but it raises an additional problem. Newtonian physics had asserted an absolute spatial reference frame and an absolute time-series. It was just this axiom which special relativity repudiated. Professor Carl Neumann in 1869 contended that some preferential body must exist in the Newtonian mechanics with some absolute basis of reference, which he called the "Body Alpha." Professor William Wilson has recently suggested that relativity theories in which the world is embedded in a five- or higher-dimensional world, have, in effect, introduced a new "Body Alpha." This new "Body Alpha" is nothing other than this n-dimensional ($n > 4$) world.[80] Professor Wilson has formulated the suspicion that was voiced by other mathematicians, philosophers, and physicists, that an absolute spatio-temporal frame was not really eliminated by rélativistic cosmologies.

But Einstein's latest attempt (he discarded earlier

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80. 1950 "The Body Alpha: An Essay on the Meaning of Relativity" Science Progress, 38, 635-636.

ones) rejects the belief that raising the number of dimensions is a useful solution. He has further tried using complex rather than real transformations of coordinates, and believes that to be of no immediate promise.[81]

In this newest solution, finished late in 1949, Einstein proposed the substitution of the non-symmetrical g_{ik} for the symmetrical g_{ik} of general relativity. Thus,

$$g_{ik} = g_{\underline{ik}} + g_{\underset{\vee}{ik}} \quad ,$$

where the bar under the covariant indices denotes a symmetrical tensor, and the $\vee$ indicates a real or purely imaginary skew-symmetric tensor. The substitution leads to the interpretation that "the electromagnetic field has to do with an anti-symmetric tensor."[82]

Three equations then suffice to give a system of attractive simplicity:[83]

$$R_{ik} = 0$$
$$g^{ik}_{+-;l} = 0$$
$$\Gamma_i = 0$$

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81. 1949 "Autobiographisches" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 90. Translated by Paul Arthur Schilpp, op. cit., 91.
82. Albert Einstein, op. cit., 90. Translated by Paul Arthur Schilpp, op. cit., 91. Schilpp's translation is used in this quotation. Einstein's term, schiefsymmetrischen, suggests that "skew-symmetric" would be a preferable translation.
83. 1950 The Meaning of Relativity, fourth edition, 140-141.

In this system $R_{ik} = 0$ replaces the field equation $G_{\mu\nu} = 0$ of general relativity. $g^{ik}_{+-;l} = 0$ and $\Gamma_i = 0$ govern the laws of the parallel transport operation. A fourth condition which may be added is that[84]

$$w_{;1} = 0 \quad ,$$

expressing the fact that the vanishing of the absolute derivative of a tensor entails the vanishing of its associated tensor density.

The full consequences of this new Einsteinian theory are not yet apparent. Professor Infeld has observed that

> Einstein's new theory is fully a unitary theory. In it only the field appears, no sources of the field. The existence of matter will have to be deduced from the field equations by finding solutions that represent great concentrations of the field.[85]

The question as to whether this new state of affairs is satisfactory lies open. Certainly Infeld's interpretation entails some consequences which philosophers, rightly or wrongly, will have difficulty in approving.

Einstein appraised the probable significance of his new unified field theory as representing a "fair probability of being found valid, if the way to an exhaustive description of physical reality on the basis of the continuum turns out to be possible at all."[86]

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84. Albert Einstein, loc. cit.
85. Leopold Infeld. 1950 "On Einstein's New Theory" The American Scholar, 19, 431-432.
86. "Autobiographisches" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 92. Translated by Paul Arthur Schilpp, op. cit., 93.

Attempts to wed quantum mechanics to relativity have been few in number, and its possibility in terms of field theories has been, in most quarters, severely doubted. Some sort of unification must, however, be achieved if the two divergent branches of physical science are to describe a common universe.

Eddington was one of the earliest to consider the problem and again applied his epistemological and axiomatic methods. Assuming Dirac's linear wave equation of the electron to be an adequate description and the validity of Weyl's relativity of gauge, Eddington postulated a four-dimensional hypersphere of uniform curvature in an Euclidean space of five dimensions.[87] It would then be this hypersphere which represented the physical continuum of events. A particle would not be modelled on classical lines, but would be a "conceptual entity whose probability distribution is specified by a wave function."[88]

The possibility of unification of the principle of invariance with quantum mechanics appeared to be the crucial issue to Professor Max Born in 1938.[89] It was apparent, he argued, that the molar gravitational laws could not be considered applicable to ultimate particles.

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87. 1936 Relativity Theory of Protons and Electrons, 55.
88. Ibid., 8.
89. 1938 "A suggestion for unifying quantum theory and relativity" Proceedings of the Royal Society of London, A, 165, 291-303.

Professor Born then defined a metric tensor γ^{kl} of a subatomic p-space, and presented equations analogous to those of general relativity:

$$P^{kl} - \tfrac{1}{2}(P + 2\lambda')\gamma^{kl} = -k\, T^{kl} \quad .$$

The corresponding model would be a closed hyperspheric momentum space.

Dirac's cosmological essay mentioned earlier (page 129 of this thesis) contained an interesting note with respect to the problem now at hand. General relativity, he reasoned, could not be expected to be valid in the cosmological model he had proposed since the gravitational "constant" would vary with the cosmic epoch, as the other natural constants would.[90]

In the application of relativistic methods to quantum phenomena, with the exception of particles with zero spin, no really valid appraisal of its position can be made, asserted M. H. L. Pryce.[91] Yet he attempted a unification of the special theory and quantum mechanics on the grounds that perhaps a definitional approach may be fruitful. Again, the continuum adopted was that of flat space-time.

It is not clear that Einstein's new theory will have

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90. 1938 "A new basis for cosmology" <u>Proceedings of the Royal Society of London</u>, A, <u>165</u>, 206.
91. 1948 "The mass-centre in the restricted theory of relativity and its connexion with the quantum theory of elementary particles" <u>Proceedings of the Royal Society of London</u>, A, <u>195</u>, 69.

solutions which might correspond to the "elementary particles" of quantum mechanics. Nor is it apparent how field theories, based as they are on a continuous, deterministic foundation, will be able to explain the discontinuous actions of elementary particles in a world where it seems that true indeterminism has reign.

Since his earlier excursion into producing a unification of the two apparently divergent branches of physical theory, Professor Born is more inclined to accent the statistical nature of physical laws. In his 1948 Waynflete Lectures, he said of the belief in the harmony of nature,

> This belief has played a considerable part in the development of theoretical physics--remember Maxwell's equations of the electromagnetic field, or Einstein's relativity--but how far it is a real guide in the search of the unknown or just the expression of our satisfaction to have discovered a significant relation, I do not venture to say.[92]

Sir Edmund T. Whittaker has pointed out that "The appeal of Relativity has been further weakened by the growing doubt as to whether continuous differential equations in four-dimensional space-time can possibly provide a solution of some of the problems of quantum theory."[93]

Professor Rosenfeld opined, "...if we choose to

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92. Natural Philosophy of Cause and Chance, 124.
93. 1950 Review of Albert Einstein: Philosopher-Scientist, Scientific American, 182, 58. This doubt has been further emphasized by Sir Edmund Whittaker throughout supervisory conferences, 1949-1951.

indulge in dreams of formal beauty, we must realise that we cut ourselves off from the 'solid ground of Nature.'"[94]

But Professor Einstein, the father of relativity and a substantial contributor to the foundations of quantum mechanics, is convinced that the attempt must continue in terms of field theory.

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94. 1950 "Professor Einstein's Dilemma" The Listener, 44, 824. The article is the script of a broadcast in the Third Programme by Professor Rosenfeld.

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CHAPTER FOUR

THE DATA AND METHODS OF RELATIVITY

Physical science, cognizant of the boundaries of its specialization, has always given the warning to philosophers that scientific reality may not necessarily be the metaphysical reality they are seeking. Metaphysicians, on the other hand, have as one of their objects, the unification, if any be possible, of the results of the more detailed inquiries into the nature of the universe, including entities not considered by the natural sciences. There is then the investigation of whether any metaphysical status is to be attached to the elementary data of those sciences. Even if no ultimately real status attaches to those data, do they hold any implied suggestions for metaphysical method, of which analogy is one of the chief constituents?[1] There is also the possibility, which must be seriously considered, as to whether the whole of metaphysics might not be the summation of the self-criticisms of all the

1. The method of analogy has been one which Thomists have consistently adopted in metaphysics. Probably the most exhaustive and fruitful non-Thomist inquiry into the use of analogy in metaphysics has been that of Professor Dorothy M. Emmet. 1945 The Nature of Metaphysical Thinking.

more special sciences.[2] The problem will be considered in greater detail in Chapter XIII.

When, therefore, science itself rejects, or at the very least, throws serious doubts upon its fundamental data of venerable antiquity, metaphysicians must, of necessity, be actively interested in the new implications.

The revolution which had been brewing for a quarter of a century had reached active expression in the relativistic writers. Minkowski was certainly among the first to realize that the die had been cast. His 1908 address to the Assembly of German Natural Scientists and Physicians in Köln was the occasion of the famous prediction: "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."[3] Minkowski's "independent reality" must not be interpreted to mean "independent metaphysical reality." It is certain that for the majority of the relativistic physicists there was a totally new frame of reference for their calculations.

On the question of the metaphysical meaning of the new

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2. For an excellent discussion of this possibility see Professor Herbert Dingle's James Scott Lecture to the Royal Society of Edinburgh for 1948. 1949 "The Nature of Scientific Philosophy" Proceedings of the Royal Society of Edinburgh, A, 62, 400-411. Sir Edmund T. Whittaker has indicated his agreement in large part with Dingle's thesis, in a supervisory conference, 1951 February 19.
3. 1909 "Raum und Zeit" Physikalische Zeitschrift, 10, 104.

space-time, perhaps Samuel Alexander was the first to comment. Alexander considered that independent reality to have been conclusively established in a metaphysical sense, and erected his famous philosophical system on this identification. Total Space-Time, the synthesis of all perspectives of space-time holds, therefore, an absolute significance.[4] From the realm of Space-Time there was a "nisus" towards self-transcendence; successive stages of realization were, for the level immediately transcended, "Deity." Alexander's work will receive more careful consideration in Chapter VII.

In spite of his unguarded ascription of independent metaphysical reality to space-time, Alexander nevertheless performed for post-relativistic philosophers a task comparable in meaningfulness to that of Bradley a little over a decade earlier. In no sense does that mean that the views of those two writers is to be identified. Furthermore, the 1916-1918 Gifford Lectures formed the first metaphysical attempt in the "grand style" which England had produced for many years. Despite a highly questionable initial groundwork, then, Alexander had performed a valuable service for metaphysics.

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4. Space, Time, and Deity, 1, 91. Reference is to pagination in the 1927 (revised) edition. The first edition appeared in 1920.

In 1919 a symposium headed by Alfred North Whitehead considered one of the questions of fundamental importance for philosophers and physicists in the new era: In what sense (if any) are time, space, and material the ultimate data of science? Inasmuch as the unique contributions of Whitehead on relativity and their implications will be considered in the following two chapters, only a summary of his 1919 memoir will be of importance at this juncture. His fundamental thesis at this stage of his development was that events are the fundamental physical data.[5] Time, space, and material are only abstractions from this new fundamental entity.

With the assertion that time, space, and material are abstractions from fundamental physical data, Sir Oliver Lodge agreed.[6] However, he insisted, that does not make them any less real; they represent the most convenient means of explaining force and motion, which he held to be the primary data of science.[7] The choice of the word "force" is an unhappy one, and the substitution of the word "field" might have been desirable. However, in view

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5. "Symposium: Time, Space, and Material, Are They, and if So, in What Sense, the Ultimate Data of Science?" _Aristotelian Society, Supplementary Volume II: Problems of Science and Philosophy_, 44-57.
6. 1919 "Symposium: Time, Space, and Material, Are They, and if So, in What Sense, the Ultimate Data of Science?" _Aristotelian Society, Supplementary Volume II: Problems of Science and Philosophy_, 58.
7. _Ibid._, 59.

of the fact that Lodge held no brief for the principle of relativity at that time,[8] perhaps he deliberately avoided the use of the word "field." Apparently he still thought in terms of his own 1894 memoir on "Aberration Problems," for he concluded, "I conceive that ultimately all the properties of the material universe will be expressible in terms of the fundamental and omnipresent ether of space."[9] Here, then, was a scientist of note who insisted that relativity had by no means necessitated a rejection of the fundamental physical data.

The contribution of Mrs. Adrian Stephen was couched in terms which Bergson might have used. The concise and pointed thesis was that "material is the ultimate datum for science, and space is an a priori form imposed upon those objects. With time science is powerless to deal."[10] In reviewing Mrs. Stephen's paper, Broad suggested a word of praise which could scarcely be better worded: "Mrs. Stephen does Bergson better than Bergson himself."[11]

The theory that conceptions of space, time, and material are rooted in complex physiological processes in

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8. Ibid., 62.
9. Ibid., 66.
10. Stephen. 1919 "Symposium: Time, Space, and Material, Are They, and if So, in What Sense, the Ultimate Data of Science?" Aristotelian Society, Supplementary Volume II: Problems of Science and Philosophy, 87-88.
11. 1920 "Critical Review" Mind, new series, 29, 233.

the percipient alone was maintained by Henry Head.[12] It is perhaps not too much to expect that some day a précise sensual pattern-physiological correspondence may some day be made, but then the problem of the external world remains unsolved. Even if the present data of time, space, and material are physiologically defined entities, the added news of this conditioning throws no further light on the extra-percipient entities which science would then describe. Necessary though it may be to remember that physiological processes are inextricably associated with perceptions, there seems to be no resultant indication of a clarification of the nature of the "external" world.

The implied result of the relativity writings was that time, space, and material, in their classical significance, were not the ultimate data of science. Time and space, as independent and absolute reference frames, had been abolished. When dealing with terrestrial mechanics, it is true, the classical forms of analysis still represented the simplest ones. However, it was understood that the classical forms were not strictly correct.

As fused space-time, in which the two formerly independent frames could be treated homogeneously, they served

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12. 1919 "Symposium: Time, Space, and Material, Are They, and if So, in What Sense, the Ultimate Data of Science?" <u>Aristotelian Society, Supplementary Volume II: Problems of Science and Philosophy</u>, 77.

to explain the operation of the gravitational field, and promised to throw light on the operation of the electromagnetic field. By explaining the operation of fields, therefore, the metric of space-time could stake a stronger claim to being "ultimate data" than its absolute space-and-time counterpart. Nevertheless, irregularities in the spatio-temporal continuum merely indicated the presence of discrete entities, on whose nature the continuum did not shed any light. Space-time could only hope to explain the interaction of the effects apparently produced by the entities with which its irregularities were associated. In the sense, then, of providing a descriptive framework for physical effects, space-time could present a reasonable claim to being an ultimate datum of science.

The metaphysical reality of space-time is highly questionable, and perhaps no really satisfactory answer can be expected. Consider, however, events happening in the physical universe. The events are defined to be happenings or persistences in a four-dimensional hypervolume of space-time. If there is to be anything metaphysically real as a result of relativity theory, it must be these various happenings. But this means the postulation of something in addition to the space-time metric: the material particles themselves, or the source of the field. The relation of space-time to these happenings is not one of contributing to their being qua a happening, but as valuable only in the sense ofdefining the boundaries or the operations of the effects of these happenings. Accordingly, a space-time would have at most only a derivative metaphysical status that would not qualify for admission to the level of reality. That thing which does the "eventing" would seem to be more real. This notion will be more highly developed by Whitehead in his philosophy of organism.

On the question of material as an ultimate datum of science, relativity had a few corrections to present to classical physics. By finding mass and energy to be interrelated, two previously independent (though associated) entities were united. Concerning the ultimate nature of "material" relativity had little news of consequence. The crucial question lay most likely at the feet of quantum mechanics. For relativity, material, with its gravitational and electromagnetic connotations, had given way to the concept of "particle," and more precisely, "event-particle." There were reasons to believe that material in the new sense of "event-particle" was to be considered one of the ultimate data of physics. Furthermore, so far as the new physical revolution was concerned, a monistic hierarchy among these particles would be preferable to a pluralistic one.

Of the various philosophical analyses derivative from the introduction of relativity theory, by far the most comprehensive and significant was that of Whitehead. His former collaborator on Principia Mathematica, however, had many useful notes to suggest.

Analyzing the problem in 1927, Bertrand Russell expressed the opinion that electrons, protons, spatial entities, etc., were really complexes of events; the events were metaphysically more primitive.[13] This statement must be regarded as replacing a view of a dozen years before to the effect that a theory of matter requires both space-corpuscles and time-corpuscles for its exposition.[14]

In classical Newtonian physics, both space and time had an absolute significance; there was one external frame of space and time which had a preferential status. The advent of the relativity theory denied this absolute status of space and time. Units of length and time were no longer rigidly defined. A measuring rod suffered under a contraction with increased velocity, and there was no reason to assume that any measuring rod was at rest. The special theory of relativity had assured the relative nature of measurements, and both length and time-extension depended on the observer's station.

13. The Analysis of Matter, 9.
14. 1915 "The Ultimate Constituents of Matter" The Monist, 25, 403.

Furthermore, because no observer had the power to designate his own or any other special reference frame as "at rest," no one or combination of spatio-temporal frames could place a legitimate claim to being the "absolute" or even the "preferred" frame of reference for the universe's operation; Professor Neumann's "Body Alpha" revived by Professor Wilson could not exist.

It will be helpful to distinguish between the space-time metric of relativity and the absolute space and time (or space-time) which is used to suggest a description of the size and the temporal activities of the universe. The latter might be referred to as "cosmological space and time (or space-time)."

The abolishing of all absolute significance of spatio-temporal frames was an act which met with open distrust in the camps of the metaphysicians. In the first place, if there is a metaphysically real world, why should there not be one description of it which should have a "cosmological space-time" significance? It may be true that an observer within the system would be subject to the spatio-temporal laws of relativity. That, however, might not mean that the total event which is the universe at a given duration of time does not describe an absolute event which is capable of having definitely delineated cosmological boundaries. The radii of the Einsteinian and the de Sitter universes had been calculated, and were expressed as absolute spatial functions. Furthermore, the universes moved along a time-axis, although infinite in extent. Milne's kinematic relativity explicitly asserted the unique nature of the time experience,[15] and offered two time-functions and two spatial descriptions of the extent of the universe. If, then, the physical cosmologies advocated by the relativists best prepared to suggest them, in essence referred the universe to a cosmological spatio-temporal demarcation, why should philosophers believe that relativists had really taken their own views on space-time seriously in the last analysis?

Kurt Gödel has recently criticized the non-empty cosmological models on the ground that the local times of all observers do merge into one objective cosmological time.[16] His own negative-λ solution, he urged, are not subject to this criticism. Einstein, in considering Gödel's suggested solution, admitted the existence of the problem in his own mind at the time of the construction of general relativity, and admitted the truth of the troublesome identification which Gödel criticized.[17]

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15. 1943 "The Fundamental Concepts of Natural Philosophy" Proceedings of the Royal Society of Edinburgh, A, 62, 19.
16. 1949 "A Remark About the Relationship Between Relativity Theory and Idealistic Philosophy" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 560.
17. 1949 "Remarks Concerning the Essays Brought Together in This Cooperative Volume" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 687-688.

It is therefore not surprising to find Alexander, in his Gifford Lectures, saying of relativity that "It leaves us Space-Time in itself as a total from which perspectives are selections; and therefore in that sense absolute and independent of observers."[18]

Kerby S. Miller likewise discovered in 1920 what he thought to be a logical contradiction at the base of relativity. Miller believed the principle of relativity to be capable of exposition in two forms. One form is the principle asserted "absolutely": "The principle of relativity presumes actual space to change with the expansion of a solid."[19] When asserted "relatively" the principle simply states the relativity of "our criteria of magnitudes, not of space itself."[20] Because the perception and mental visualization of space are, in effect, of a fixed nature, Miller believed that the principle of relativity could not be asserted "absolutely," but only "relatively."

This argument is a restatement of the philosophical desire to preserve an absolute space-time, and is derivative from an understandable confusion of the statements of relativity. In the first place, as Miller conceived the absolutely asserted principle of relativity, he was stating

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18. Space, Time, and Deity, 1, 91.
19. 1920 "The Logical Necessity of a Constant in the Concept of Space" The Journal of Philosophy, Psychology and Scientific Method, 17, 438.
20. Ibid., 438-439.

not the principle of relativity (either special or general), but a notion derivative from the cosmological models of de Sitter, Einstein, and most of the exponents of expanding universes, with the notable exception of Milne. It is true that the metric of space-time in general relativity varies with the size of the irregularity around that metric. However, it was not the larger size, but the greater concentration of mass, in the companion of Sirius which was held to be responsible for the greater deflection of light rays grazing its surface.

Indeed, it was one of the points of dissention introduced by Milne's kinematic relativity that space-time itself did not expand. The expansion of space-time, he insisted, was a meaningless assertion. It was only the concentrated bits of matter at the surface of the sphere which moved radially outwards, causing an enlarged universe. Indeed, every particle inside the kinematic universe was directed outwards from the center of that universe, but the metric did not expand as a consequence.[21]

It cannot, of course, be denied that Milne did reintroduce a meaningfulness for an absolute interpretation of cosmological space and time with his kinematic universe in t-time, and with the preferred t_0 of 2×10^9 years. It

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21. E. A. Milne. 1935 Relativity, Gravitation, and World-Structure, 131.

may further be added that the majority of the cosmological models as they stand seem to be as easily pictured in an absolute background of space-time or (in kinematic models) space and time, as within a totally relativistic frame.

Sir Edmund Whittaker has pointed out that in Milne's system there is no way for an observer to locate himself in the universe with reference to this cosmological space-frame. To this type of postulate he has given the happy name, "Postulate of Impotence."[22] This particular postulate of impotence has been extended to include the further impossibility of telling exactly where the observer is located in the time-scale. Other postulates of impotence suggested by Sir Edmund Whittaker might be added as being basic to current cosmological theory: that of the impossibility of detecting absolute uniform translatory motion of a system from within that system, and that of the impossibility of a continuous identification of an electron. Such postulates, in the opinion of this writer, represent a tremendous addition to cosmological theory, and could be profitably examined by metaphysics.[23] Metaphysics itself (if it be distinct from Dingle's sciences-summation) seems

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22. 1949 From Euclid to Eddington: A Study of Conceptions of the External World, 59.
23. Professor A. D. Ritchie, using this reference, engaged in just such an exercise in the Senior Honours class in Logic and Metaphysics at Edinburgh University, 1951 February 23. The conclusion reached is substantially identical with that of this thesis.

to have certain Postulates of Impotence which it might be helpful to codify.

Of course, certain postulates would qualify for inclusion only in certain systems. But others seem to have an air of persistent unattainability in any system. Among possible applicants for inclusion may be (1) the impossibility of attaining a perfect metaphysical description of the universe, (2) the impossibility of perfect knowledge devoid of subjective experiencing, (3) the impossibility of demonstrating the justification for the metaphysical elementary constituents (substance, organisms, the Given, or whatever they be) in any given metaphysical system, and (4) the impossibility of experiencing other people's experiences.

One of the more telling criticisms of orthodox relativistic cosmology has been that it introduced a questionable interchangeability of space and time. This criticism is not peculiar to philosophers, and was one of the difficulties leading to kinematic relativity. Bergson's entire philosophical work is evidence that, for Bergson at least, the flowing time was an undeniable constituent of human experience. The independence of time in some sense from space was one of the points of divergence between Whitehead's philosophy of nature[24] and that of some of the

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24. See especially Whitehead. 1920 The Concept of Nature, 49-73.

relativistic cosmologies. Milne further asserted that, for observational experiences, each individual adopts a "world-wide" Newtonian time-scale.[25] Eddington also sensed this difficulty and at one time[26] introduced a pentadic coordinate system which he felt accounted in some way for the uniqueness of time. Of these five mutually perpindicular coordinates, no more than three could have the same real or imaginary character. Hence, there was an opportunity, perhaps remote, for providing a different meaning to time. However, this device was little more successful than early attempts of the post-special relativity era of using an imaginary time-scale, and no more successful in defining the nature of time and space as distinct. Professor Born has pointed out the fact that the interchangeability of space and time is true only in a very restricted sense.[27]

Beginning with Hubble's suggestion that the spiral nebulae may be receding with a fairly definite velocity, Dirac in 1938 provided a more natural explanation of the distinctive character of time. The fact that such a recession velocity might suggest a cosmological model wherein a material particle had a "natural velocity" led Dirac to

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25. Milne, <u>op. cit.</u>, 44.
26. 1936 <u>The Relativity Theory of Protons and Electrons</u>, 53-75.
27. 1950 November 14. Lecture at Edinburgh University.

postulate a preferred time-axis for each point-particle.[28] This preferred time-axis would be that one with respect to which the matter in the immediate neighborhood of the point would be at rest. Measurement of intervals along this axis would provide an absolute time-measure which he called an epoch. However, in view of the postulates of impotence respecting location of an observer in time and the detection of absolute rest velocity, such a definitional suggestion would expose itself to a highly pointed attack.

The problem of accounting for the distinctive nature of time remains unsolved in most systems, although both Milne and Whitehead can lay claim to providing reasonable accounts.

One of the problems arising from the relativity writings was the status and interpretation of the critical velocity _c_. It was apparent, by definition in the special theory, that it was an absolute quantity equivalent to the velocity of light _in vacuo_--3 x 10^{10} cm/sec. The general theory demonstrated that, in the presence of a gravitational field, its velocity was altered. Einstein asserted that c had a further physical significance: that it was a limiting velocity which can be neither reached nor exceeded

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28. "A new basis for cosmology" _Proceedings of the Royal Society of London_, A, _165_, 199.

by any material particle.[29] Milne, on the other hand, developed a cosmological theory in which the indefinitely large number of particles at the outer surface of the spherical t-time universe possess that velocity. Whitehead placed no absolute meaning on the velocity.[30] A further doubt was cast by de Broglie, who observed that c could be considered a limiting velocity only for time-signalling mechanisms. Einstein has recently suggested that, by appropriate transformations to eliminate the dimensions of space and time, the velocity c is relegated to the status of an apparent, rather than a real, universal constant.[31] Indeed, he has postulated that most likely the laws of nature are such as to contain no such arbitrary constants in their final formulation,[32] a trend not uncommon in recent theories. This question, too, remains unsettled.

However, the postulation of the nature of c by the special theory raised another disturbing consequence which has received a tremendous amount of discussion. That is the problem of simultaneity. The special theory implied

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29. 1920 Relativity: The Special and the General Theory: A Popular Exposition, third edition, translated by Robert W. Lawson, 36.
30. 1920 The Concept of Nature, 131.
31. 1949 "Autobiographisches" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 60. Translated by Paul Arthur Schilpp, op. cit., 61.
32. 1949 "Autobiographisches" op. cit., 62. Translated by Paul Arthur Schilpp, op. cit., 63.

that no absolute significance could be placed upon the concept of simultaneity.[33] Events which were simultaneous in one time-system would not necessarily be so in another. Weyl, in his 1917 lectures, hailed this innovation as the achievement which would warrant the placing of Einstein's name in the category of inventive discovery inhabited by Copernicus.[34]

In denying an objective meaning to simultaneity, there was a philosophical difficulty. It was one which hinged on the relation between measured time in relativity and experienced time for an observer. Wildon Carr summarized the trouble in his opening contribution to the Aristotelian Society symposium in 1923:

> The rejection of simultaneity involves a paradox. In regard to time measured any two events which for observers in one frame of reference are simultaneous, for observers in other frames are before and after. In regard to time lived, they are identical for the lives in all systems.[35]

However, there is no real paradox; in the first place, relativity had demonstrated how, because of the differences in the relative motion of observers, light signals

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33. Albert Einstein. 1905 "Zur Elektrodynamik bewegter Körper" Annalen der Physik, (4), 17, 897.
34. Space-Time-Matter, fourth edition, translated by Henry L. Brose, 174.
35. "Symposium: The Problem of Simultaneity: Is There a Paradox in the Principle of Relativity in Regard to the Relation of Time Measured to Time Lived?" Aristotelian Society, Supplementary Volume III: Relativity, Logic, and Mysticism, 25.

from two events would not possibly in general reach both observers at the same time in both systems. Furthermore, the life-experiences of any individual observers are such that the small difference in their relative motion would likely not be sufficient to cause any appreciable difference within the apparent simultaneity of distant events according to two observers.

However, the problem of constructing a public time continued to be troublesome, and in 1927 J. L. Synge defended objective simultaneity once more. Postulating a rigid Euclidean reference-frame which would be isotropic with respect to light-propagation, he demonstrated that the time of any event could be uniquely determined,[36] as Milne asserted at a later date. Consequently, simultaneity would be a transitive experience. The divergence between these treatments and Einstein's occurs at the introduction of an isotropic Euclidean space-frame with respect to light-propagation. The question is therefore unsettled, and the two camps disagree as to its answer.

Perhaps the guiding spirit of relativity may be described as the conviction that the material properties of the universe could be described in terms of the spatio-temporal metric. The notion was heralded by Minkowski,

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36. "Time Measurement in an Isotropic Space Frame" Proceedings of the Royal Irish Academy, A, 37, 110.

who is more important to the theory of relativity than many writers have conceded. In 1908 he prophesied,

> The whole universe is seen to resolve itself into world-lines, and I would fain anticipate myself by saying that in my opinion physical laws might find their most perfect expression as reciprocal relations between these world-lines.[37]

There is an interesting contrast between this phraseology and that of Sir Oliver Lodge, who, it will be remembered, was not sympathetic with relativity theory in 1919: "I conceive that ultimately all the properties of the material universe will be expressible in terms of the fundamental and omnipresent ether of space."[38] Weyl offered a guiding corrective, which might have given rise to a violent philosophical controversy, by referring to the spatio-temporal framework as giving form to the phenomena in contrast with the classical view.[39]

Perhaps one of the most controversial basic questions excited by the relativity era was just this attempt to geometrize the physical influences being described. Thus the metric itself told the story of the gravitational field, and a generalized or extended geometry has been

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37. 1909 "Raum und Zeit" Physikalische Zeitschrift, 10, 104.
38. "Symposium: Time, Space and Material, Are They, and if So, in What Sense, the Ultimate Data of Science?" Aristotelian Society, Supplementary Volume II: Problems of Science and Philosophy, 66.
39. Space-Time-Matter, fourth edition, translated by Henry L. Brose, 227.

employed to account for the electromagnetic field. As was observed in Chapter III (see page 144 of this thesis), Professor Infeld believed the new unification to be valuable in its removal of the dualism (geometrical description of gravitation--physical description of electromagnetism) of general relativity. That Einstein's geometrization of gravitation was the most valuable gain in knowledge by general relativity is suggested by Professor Finlay-Freundlich.[40] But with this assertion Whitehead disagreed. He held that the attempt must be made to describe physical influences in terms of physical qualities, as Poynting had believed. Sir Edmund Whittaker has recently suggested an important principle which might give pause to the desire to make geometries the vehicle of the physical world by pointing out the "fact, which is of capital importance throughout natural philosophy, that a correct *mathematical* solution of a phenomenon does not necessarily furnish the correct *physical* description of the phenomenon."[41] Because it avoids this danger, Whitehead's relativity may well repay further consideration from the mathematical physicist. It is not suggested that Whitehead's relativity is the most correct, but by avoiding certain basic philosophical difficulties, as well as for

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40. 1950 August 28. Letter to the author.
41. 1946 *Space and Spirit: Theories of the Universe and Arguments for the Existence of God*, 56-57.

its own merits (discussed in Chapter VI), it seems to be entitled to a more persistent examination.

The scientific controversy is extended to include the question whether a description of the properties of the material universe in terms of differential equations presupposing a continuous background is possible. The present impasse arises from quantum mechanics, whose results indicate that such a description of subatomic phenomena is impossible.

The problem leads naturally to another impasse in recent physics, with quantum mechanics on one side and relativity physics on the other. The problem is that of determinism. Perhaps Russell was the earliest post-relativity philosopher to support the quantum indeterminism in a cogent argument. "I do not know," he insisted, "whether there is a persistent entity, but I do know that my experiences can be explained without assuming that there is."[42] Eddington had in 1920 suggested that certainly the most important law in the external world was the law of atomicity,[43] but this admission should probably be taken in conjunction with his statement of the same year to the effect that there was an intrinsic orderliness in nature.[44]

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42. 1927 An Outline of Philosophy, 125-126.
43. "The Meaning of Matter and the Laws of Nature According to the Theory of Relativity" Mind, new series, 29, 156.
44. Space, Time, and Gravitation: An Outline of the General Theory of Relativity, 54.

The two statements, when reconciled especially in the light of field theories implied by the last statement, would not represent a case for indeterminism. Furthermore, it was not until after Heisenberg's announcement of the Principle of Indeterminacy that the situation was more completely clarified.

Professor Born has quoted from a letter which he received from Einstein, and which localizes the divergence in regard to determinism among contemporary physicists. Dated in December of 1947, Einstein's letter reads,

> I see of course that the statistical interpretation has a considerable content of truth. Yet I cannot seriously believe it because the theory is inconsistent with the principle that physics has to represent a reality in space and time without phantom actions over distances. I am absolutely convinced that one will eventually arrive at a theory in which the objects connected by laws are not probabilities, but conceived facts.[45]

Certainly Einstein has the ordinary experience of men in his favor: events in the external world apparently occur in a continuous sequence. Furthermore, the individual has the impression of a continuous existence, at least in his conscious states. It is not clear whether a metaphysics dominated by indeterminism would be able to explain adequately the macroscopic realm. For each theory there remains

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45. Quoted in 1949 Natural Philosophy of Cause and Chance, 123.

the duty of explaining the phenomena of the rival realm. Quantum mechanics holds the statistical method to be the only possible answer; relativity theorists have adopted various deterministic means of explanation. The method intrinsic to Whitehead's treatment belongs to the side of the defenders of determinism and field theory.

The problem of perception presents itself. Early after the publication of the general theory, de Sitter observed that what is perceived would be the intersections of world-lines.[46] But then the question called into consideration is whether the point-event representing those intersections can be existentially perceived. Apparently a certain extension in both space and time is necessary in order for perceptual experience to take place. It might conceivably be urged that a certain hierarchy of intersecting world-lines need to intersect in order that perception can take place. This sort of postulate is reminiscent of Whitehead's linear objective reals of the 1905 memoir. But the same difficulty arises. Are these intersections really the sensed data? An affirmative answer is by no means apparent.

Inasmuch as the laws of the physical world and the laws of perception refer to a common set of entities, they

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46. 1916 "On Einstein's Theory of Gravitation, and its Astronomical Consequences--I" Monthly Notices of the Royal Astronomical Society, 76, 700.

must be in some sense related. This fact would lead, insisted Russell in 1927, to an interpretation of physics which would be more idealistic than previously, and to an interpretation of perception which would be more materialistic than before.[47] Just such a solution appeared in Whitehead's philosophy of organism.

Philipp Frank has, however, thrown doubt upon carrying this method of thought too far. He has insisted that one of the greatest causes of philosophical misinterpretations of physical science is that there is a tendency to make statements about observable processes in the external world assertions about a real metaphysical world.[48] Such a statement in the tradition of Mach would render meaningless any metaphysical propositions of any sort. In such a case, however, metaphysics as a philosophical inquiry would be succeeded by scientific cosmology. Furthermore, inquiries indispensable to metaphysics would be relegated to inquiries such as psychology and biology. Eventually another field known probably as the philosophy of nature would be necessary to give a coherent account of the various results of its subsciences. This is the situation as Dingle believes. In the opinion of this thesis, however, another step, that toward metaphysics, or an

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47. The Analysis of Matter, 7.
48. 1949 Modern Science and Its Philosophy, 160.

equivalently directed inquiry, is necessary.

One of the greatest philosophical misinterpretations which quickly followed in the wake of relativity was the assertion that relativity presupposed an idealistic interpretation of the universe. Probably the outstanding cause of this misunderstanding lay in the confusion of the connotations of "observer." The shadings are immediately removed by the substitution of "observer's body" for "observer." No harm is done to relativity theory, and no idealistic implications can then be attached to that potentially dangerous point.

However, the Aristotelian Society again sponsored a symposium on relativity, this time in 1922 on the idealism-relativity relation. The thesis that there was a necessary connection between the two was maintained by H. Wildon Carr[49] primarily on very shaky grounds. Einstein's theories, Carr insisted, are based on the recognition that the phenomena of the physical world are presented to an observer only in the form of sense-experience. Further, these entities are inseparably related to the minds perceiving them. This state of affairs he found to be consistent with neo-idealism, which he defined as "the philosophical standpoint that reality in its fundamental and

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49. "Discussion: The Idealistic Interpretation of Einstein's Theory" Proceedings of the Aristotelian Society, 22, 123-127.

universal meaning is mind or spirit."[50] Neo-realism he defined to be the view that

> knowledge requires us to presuppose existence, and that in some sense a universe exists in space and time, the entities within which are discoverable by minds, which are themselves accorded a place therein on equal terms with the entities they discover.[51]

Now there is no reason in the entire scientific literature of relativity for believing that any of its conclusions consistent with neo-idealism (or any kind of idealism) would not be equally consistent with neo-realism (or any kind of realism). Reality, either observable or metaphysical, is rendered no more idealistic by relativity. If there is to be a selection between realism and idealism the impetus must come from another region, and not from physical cosmology. Although the metaphysics will provide the background for physical cosmology, the physical universe of relativity gives no preference between realism and idealism.

The three other contributors to the discussion[52] agreed that there was no necessary connection between relativity and either the older or newer idealisms.

Wilhelm Wirtinger, who wrote of generalized geometry

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50. Ibid., 124.
51. Ibid., 124.
52. T. P. Nunn, op. cit., 127-130. A. N. Whitehead, op. cit., 130-134. Dorothy Wrinch, op. cit., 134-138.

with a view to offering the mathematical framework for unified field theories, suggested that a reconciliation might be effected between the new geometries and the Kantian _a priori_ space and time.[53] However, again it must be insisted that although such a view may be possible, it is by no means necessary, and the connection would be difficult to demonstrate.

The methods of relativity are of great interest to philosophy. Essentially the methods are of the axiomatic-deductive rather than the inductive type. The method consists of defining certain entities having certain properties and of postulating certain relationships among those entities as valid. In practice, certain intermediate hypotheses are introduced, and the deductive consequences of the postulates and their implied relations are found. These deductive consequences, interpreted in terms of the fundamental definitions, are then tested for correspondence to observed facts and relations. In essence the theories of relativity are axiomatic-deductive systems whose consequences will lead only to a description of relations between entities. This method is exactly that which Whitehead employed in his "On Mathematical Concepts of the Material World." In that case, a set of entities were

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53. 1922 "On a General Infinitesimal Geometry, in Reference to the Theory of Relativity" _Transactions of the Cambridge Philosophical Society_, _22_, 448.

defined, and a polyadic relation of order postulated. Then deductions of the implications of the properties of the entities themselves were made. The third stage represented the selection of the group of the appropriate hypotheses or axioms. The fourth step was the examination of the deductive consequences of the first, second, and third stages.[54] The final stage was the comparison of the deductive results with the known experiential facts of the area investigated. Psychologically, Whitehead observed, the order is usually inverted.[55]

In the case of the relativity theories, a similar procedure can be traced. The entities assumed are the material point-events. The axioms introduced are the special and general principles of relativity defining the relationships between those point-events when considered in uniform or accelerated motion. Unified field theories represent a more generalized description of those laws of relations among event-particles. The final stage is the comparison of results with the observed phenomena due to the operation of the fields.

In Whitehead's 1905 memoir the comparison was between the results of his own axiomatic-deductive system and the theorems of Euclidean geometry. A further stage was the

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54. MCMW, 469-470.
55. MCMW, 470.

definition of material entities in terms of the deductive superstructure. The intermediate hypotheses necessary for the description of the actions of the material entities were only briefly indicated.

Weyl was of the opinion that when the experiential fact that the gravitational field is governed by the distribution of matter was used as one of the axioms relating the entities, it was of more importance than the principle of invariance.[56] The question of the relative importance of these principles is perhaps pointless at present. If either, when generalized to the point necessary, is shown to be more instrumental in accomplishing further unification in physics, that one may have a right to be considered more "important." It has been seen that Professor Born considered at one time that the touchstone to the unification of relativity and quantum mechanics lay in the invariance hypothesis.[57] The invariance principle also has a venerable history of providing possible touchstones in Dirac[58] and in Eddington.[59] Einstein himself has suggested that this may be the key postulate for further attempts at

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56. 1922 Space-Time-Matter, fourth edition, translated by Henry L. Brose, 226-227.
57. 1938 "A suggestion for unifying quantum theory and relativity" Proceedings of the Royal Society of London, A, 165, 291.
58. Cited in Eddington. 1936 The Relativity Theory of Protons and Electrons, 1.
59. Op. cit.

unification.[60] He has, however, also indicated that the limiting case of the pure gravitational field and its relation to the metric of space may have an additional claim to final significance.[61]

Additional applicants for inclusion as basic axioms in relativist cosmologies are the epistemological principles invoked by Sir Arthur Eddington.

That the axiomatic method derives its authority from inductive experience, Einstein has denied. An important point is that he has also denied that concepts originate from experience through the road of abstraction, but rather are free inventions which are then compared with the experience. This "free invention" conception of scientific method is the object of much of the attack of the quantum physicists.

The question as to the comparative usefulness of induction and axiomatics in preparing a physical cosmology constantly recurs and has had a long history. It seems that both methods are necessary for a successful elaboration of the "laws of nature." Without the inductive method, there would be a barren desert with which axiomatics

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60. 1949 "Autobiographisches" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 68. Translated by Paul Arthur Schilpp, op. cit., 69.
61. 1949 "Autobiographisches" op. cit., 74. Translated by Paul Arthur Schilpp, op. cit., 75.

would need to compare its results. Without the inductive method, axiomatics would lack direction. But without axiomatics, the results of the inductive method would be no more than a catalogue of recorded data. The two methods cannot be at war with each other; they need to complement each other in order to achieve the optimum results. Those optimum results would be the presentation of a system which would describe the full operation of the universe, and the precise chain of connection from principles to manifestation in "nature" would be apparent. Relativity theory attempted to present a system doing just that for the gravitational and electromagnetic fields. It has been partially successful, but the system is not large enough to admit an explanation of other results which have been observed.

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CHAPTER FIVE

DATA AND METHODS IN WHITEHEAD'S PHILOSOPHY OF NATURE

The publication of the special and general theories of relativity and the commentaries on them, aroused an active interest in Whitehead, who had been collaborating with Bertrand Russell on Principia Mathematica. Dissatisfied with the interpretations placed on relativity, and sensing an inadequacy in the epistemological foundations of relativity, Whitehead began a period of prolific writing on the subject. In the years between the publication of Principia Mathematica and the appearance of Science and the Modern World in 1925, he offered no fewer than fifteen articles which dealt directly with the subject, including the famous three nature-philosophy books of 1919-1920-1922. Short articles on education also appeared frequently during this period, but it was primarily one of production in the philosophy of science, and culminated in the presentation of a theory of relativity which might have been a serious rival to the Einsteinian theory, and which still has a strong claim for acceptance.

Observing, then, difficulties mentioned above, Whitehead concentrated his attention on examining certain postulates in the axiomatic-deductive method of relativity.

That he was concerned also with the empirical verification of the consequences of the axioms is shown by an early warning: "Our problem is, in fact, to fit the world to our perceptions, and not our perceptions to the world."[1] It is in the papers published that many of the objections raised to An Enquiry Concerning the Principles of Natural Knowledge, The Concept of Nature, and The Principle of Relativity are answered. It would have indeed been better for him to discuss those questions at greater length in the three main books of the period, but they had been considered elsewhere, and their republication would have represented much unnecessary reduplication. However, references in the three volumes to these other papers are sparse: there are none in the Enquiry, two in the Concept, and none in Relativity. Furthermore, the papers immediately prior to the three main "nature" volumes show how certain ideas in those volumes evolved; succeeding papers and Science and the Modern World exhibit the trend from science to metaphysics. The entire sequence shows a constant development of cosmology from an attempt to stabilize the philosophical foundations of relativity to an attempt at showing the derivation of the whole of the

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1. 1916 "Space, Time and Relativity" Proceedings of the Aristotelian Society, 16, 129. Henceforth this memoir will be designated in footnote references as STR.

cosmological theory from metaphysics. It is certain that Whitehead himself sensed many of the objections to the results of the various stages of his progress. Otherwise the development would seem to have culminated in his three companion volumes of the turn of the decade. The development undoubtedly reached fruition there, but it also drew into bold relief the challenge from metaphysics. A marked change was apparent in *Science and the Modern World* (1925), and the matured product was crystallized, with some few exceptions, in *Process and Reality* in 1929.

It is therefore one of the opinions urged here that Whitehead's relativity writings can be successfully considered only in conjunction with the shorter memoirs of the twenty-three year period, and with the philosophy of organism in mind.

As early as "Space, Time and Relativity" Whitehead pointed to the fact that "fragmentary individual experiences are all that we know,"[2] and insisted that it must be from this basic givenness of atomicity that a meaningful theory of the external world must be constructed. Atomic entities are thus the basic building bricks of the world. At that time he was unwilling to allow that a percipient is directly aware of a uniformity of texture as a natural background. It was only because a deductive superstructure

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2. STR, 122.

could be synthesized to describe these "atomic" experiences that a uniform background could be supposed to exist. One year later, he was willing to concede that fragmentary continuity might be an immediate fact of experience, but still insisted on the "radically untidy, ill adjusted"[3] nature of experience. In the year 1919 Whitehead reasserted that perception consisted of an awareness of events,[4] but did not explicitly reaffirm its untidy character. However, it is apparent that the swing to postulating an immediate experience of uniformity did not approach the other extreme, for in The Principle of Relativity the observation was made that without the atomicity of perceptive experience, particular problems could not be isolated.[5] By thus combining the undeniable fact of disjointed experiences with the suggestion of a background fabric of uniformity, Whitehead has escaped one of the fundamental difficulties in the postulates of many philosophical systems.

By way of further analyzing the suggested background

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3. 1917 "The Organisation of Thought" Proceedings of the Aristotelian Society, 17, 61. Henceforth this memoir will be designated as OT.
4. 1919 An Enquiry Concerning the Principles of Natural Knowledge, 68. This volume will hereafter be referred to as PNK. Awareness of events was also suggested in STR, 107, and was later repeated in The Concept of Nature, 15. The Concept of Nature will hereafter be known as CN.
5. 1922 The Principle of Relativity with Applications to Physical Science, 73. This volume will be referred to as REL.

connecting the events, in "Space, Time and Relativity" it was the changing relations between events which were perceptually distinguished,[6] thus foreshadowing the postulation of the adjectival character of matter. The situation was clarified in Whitehead's contribution to the symposium, "Time, Space and Material," mentioned in the preceding chapter. Here the awareness beyond fragmentary events is that of a temporal, durational whole of nature.[7] Further, in the passing fragmentary events there are certain repeated permanent components which might be designated by the name, "object."[8] But the ultimate fact for observational knowledge is the apprehension of an event, which occupies a duration of a "temporal slab of nature."[9] An event which is perceived is to be known as being cogredient with that one duration.[10] But for any duration there may, for instance, be the following events cogredient with it: a small child bouncing a ball, the chirping of a bird, the rumble of a distant tram, etc. On the appearance of The Principle of Relativity, Whitehead repeated that this

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6. STR, 107. Clarified to mean spatio-temporal relations in a note, 121.
7. 1919 "Symposium: Time, Space and Material, Are They, and if So, in What Sense, the Ultimate Data of Science?" Aristotelian Society, Supplementary Volume II: Problems of Science and Philosophy, 46-47. Whitehead's contribution to this symposium will be known as TSM.
8. TSM, 51.
9. PNK, 69. CN, 53. REL, 7.
10. PNK, 70.

background exhibits itself as a uniform spatio-temporal relational frame highly analogous to, but not exactly that of, general relativity.[11]

Furthermore, he was prepared to assert a dual constitution of the nature of perception. Cognizance of a recognizable recurring constituent, such as a patch of red color, would be known as cognizance by adjective. Knowledge of the interrelationships of nature via the spatio-temporal frame would be called cognizance by relatedness. The two cognizances in summation, would be equated with perception.[12] The division is almost that of the objects of mental and physical prehensions in Process and Reality.

In thus attempting to explain the natural world in terms of the sensible experiences of it, Whitehead has laid himself open to a criticism by Northrop, who believed the attempt would prove weakened because Whitehead demanded more intelligibility in sense-experience than it warrants.[13] Northrop suggested instead a two-termed epistemic correlation of the immediately sensed and the postulationally prescribed components of human experience.[14]

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11. REL, v, 14. Originally suggested in STR, 121.
12. REL, 62-64.
13. 1941 "Whitehead's Philosophy of Science" The Philosophy of Alfred North Whitehead, The Library of Living Philosophers, 3, 205. Repeated in 1946 The Meeting of East and West: An Inquiry Concerning World Understanding, 442.
14. 1946 The Meeting of East and West: An Inquiry Concerning World Understanding, 443.

Whitehead's response would necessarily be that the postulationally prescribed component is in some manner indicated in the immediately sensed component. This would not necessarily be so with Einstein. However, it is the opinion of the present writer that unless sense-awareness is found to supply some basis for the construction of a metaphysical or even a cosmological edifice, resulting superstructures will tend to be unnecessarily fictional. Whether sense experience exhibits a suggestion of underlying uniformity is apparently a point of divergence, and is not without powerful adherents on either side. But this has received implicit criticism by Einstein, who holds that to imagine concepts to originate from experience is abortive.[15] B. Mayo has recently reaffirmed the position that events are directly sensed, and that in some way there is also an experienced sense of duration.[16]

Thus, Whitehead was convinced that of the events constituting human perceptions of the external world something significant could be said as a whole, and he believed that spatio-temporal properties would be the terms of at least some of those statements. The task of science would be the refinement of concepts deriving from the

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15. 1944 "Bemerkungen zu Bertrand Russells Erkenntnis-Problem" The Philosophy of Bertrand Russell, The Library of Living Philosophers, 5, 278-291. Translated by Paul Arthur Schilpp, op. cit.
16. 1950 "Is There a Sense of Duration?" Mind, new series, 59, 71-78.

the experiential data. As the refinement increases, generalized concepts exhibiting uniformity appear, of which one example is mathematical time.

In attempting to formulate the general relations existing in a spatio-temporal frame, Whitehead in 1914 suggested a spatial relation which he called "inclusion."[17] Proceeding in a method familiar to students of Principia Mathematica, Whitehead postulated a world founded on a class of relations, which is then specialized to the relation of "inclusion." It may be that the impetus for selecting this particular relation arose from the 1905 memoir, to which he here referred.[18] Victor Lowe has also suggested that particularly the Theory of Interpoints may represent the germ of the Method of Extensive Abstraction,[19] which will be discussed later in the chapter.

In the 1914 analysis, points, lines, and surfaces were defined in terms of the fundamental relation of inclusion, and are known as material T-points, material T-lines, and material T-surfaces corresponding to those geometrical entities occupied by material entities.[20] The

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17. 1916 "La Theorie Relationniste de l'Espace" Revue de Métaphysique et de Morale, 23, 440. This memoir will hereafter be referred to as TRE.
18. TRE, 435.
19. 1941 "The Development of Whitehead's Philosophy" The Philosophy of Alfred North Whitehead, The Library of Living Philosophers, 3, 39.
20. TRE, 441.

class of objective reals is not necessarily linear, but any spatial entity at all. The important similarity to observe is that the final geometrical entities are derived from complexes of the classes of the spatial entities. The outcome was a definite preference for a relational theory of space, just as there was an implicit preference for the relational theory in "On Mathematical Concepts of the Material World."

In the symposium on "Time, Space and Material" Whitehead introduced the notion that time does not consist of a series of instants, and that Nature at an instant is an abstraction.[21] Neither corresponds to any direct knowledge on the part of an observer. Any apprehension of nature requires a discrimination of various events and an awareness of the temporal duration of Nature.

The basic facts in physical nature, the events, are connected by two fundamental relations known as "extension" and "cogredience."[22] The continuity of nature then arises purely from the extensional property of events.[23] Now when the termini of sense-awareness, called facts, possess certain characteristics they are known as "events." These characteristics are known as the "constants of

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21. TSM, 45.
22. TSM, 47. PNK, 4, 61, 86. CN, 75.
23. Quoted in TSM, 55. Further similar statements appear in PNK, 67. CN, 76. REL, 67.

externality,"[24] and are really examples of the "intermediate axioms" habitually assumed in the axiomatic method. These constants include the promise of apprehended entities to provide a definite complex of entities available for knowledge. A second constant is the assertion that the relation of extension holds. It is a derivative meaning of this constant that infinitesimal bits of space and time are abstractions from this relation.[25] The third constant is that the apprehended event is related to the complete scheme of nature by the fact that the duration of nature extends over that apprehended event. Furthermore, the duration of the percipient event is also the duration which extends over the event as apprehended. The fourth constant expresses the relation between the apprehended and the percipient event. The fifth constant is the fact that there exists a relation of a percipient event to its duration. The sixth constant expresses the fact that the fragmentary apprehended events are related to a community of nature.

With these constants of externality it would be impossible to find any fact for apprehension which does not in some sense exhibit relatedness to other facts. This indicating process is known as the "significance of

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24. PNK, 71-72.
25. PNK, 75. REL, 21.

factors."[26] Each event therefore in some sense signifies the entire structure of events contributing to the operation of nature.[27] Thus an event occupies a four-dimensional hyper-volume in the spatio-temporal continuum, which must be uniform. Within nature any given number of events will be parts of and signify, some larger event.

There is a possible objection which might be urged against this doctrine, as against Einstein's. That objection is that no knowledge of the natural universe is possible until each and every event and every member contributing to the significance of factors is known. However, Whitehead forestalled the objection by distinguishing between essential and contingent relations. The essential relations are those which give the status of an entity to a factor, while contingent relations are those which are not essential to the being of the factor qua factor.[28] It will then become the duty of science to discover essential relations within the natural universe, and to limit the sphere of contingency.[29]

By the time Whitehead was preparing a second edition of The Principles of Natural Knowledge, he was convinced

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26. REL, 18.
27. 1923 "Uniformity and Contingency" Proceedings of the Aristotelian Society, 23, 4. This paper will be referred to as UC.
28. REL, 22-23.
29. REL, 29. Cf. UC, 1.

that extension was not the fundamental relation in "Nature." Instead, it was derivative from the fundamental notion of "process,"[30] a selection which caused much displeasure on the part of many of his readers. Science and the Modern World in 1925 gave further voice to Whitehead's assertion that process was the really fundamental idea at the base of natural operations.[31]

Braithwaite's review of Science and the Modern World asserted that the new volume tended towards a monism, because every event was made to require every other event for its full existence.[32] However, as has been demonstrated, this is not something peculiar to Science and the Modern World. Whitehead had been urging such a mutual significance of events since the appearance of The Principles of Natural Knowledge. Victor Lowe has pointed out that the notion of significance is basic to the cosmology that Whitehead was expounding.[33] That this significance beyond elements must not be forgotten, even by the sciences, has been emphasized by Professor Finlay-Freundlich.[34]

Inextricably related to the mutual significance of

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30. 1925 PNK, second edition, 202.
31. Science and the Modern World, 90. The book will hereafter be denoted SMW. Page references to SMW are to any of the Cambridge University Press editions of 1932 to 1946.
32. 1926 "Critical Notice: Science and the Modern World, by Alfred North Whitehead" Mind, new series, 35, 491.
33. Victor Lowe, op. cit., 76.
34. 1950 August 28. Letter to the author.

events is the necessity for requiring the essential relations between those events to be internal.

A common view that Whitehead's work of this period was essentially critical in nature arose primarily from two sources, and is not strictly true. It derived its limited meaningfulness from Whitehead's repeated assaults on what he called the fallacy of the "bifurcation of nature." A second factor was his repeated reference to Einstein's relativity in contrast to his own. The relativity contrast will be more appropriately postponed until the next chapter. With respect to the bifurcation of nature, however, the quarrel was also with older notions which formed the basis of classical physics and pre-relativity metaphysics.

The criticism is no longer an implicit one of limited applicability and cumbersomeness, as in the 1905 memoir. It is here an explicit accusation of incoherence. As early as "Space, Time and Relativity" the rebellion clearly appeared without attaching the descriptive name connected with the fallacy exposed by The Concept of Nature. Essentially the fallacy centers in the fact that two real worlds are assumed to exist with no direct means of communication between the two. The one world is the world of the postulated scientific entity, which is the ultimate fact for knowledge. The other world is the perceived world, whose relations to the first are highly indeterminate, and which is conceived to have its reality in the perceiver's mind.[35] On the one side there is the world of "hurrying electrons," instants of time, and extensionless points of space. On the other is the world of sensed macroscopic objects, the specious present, and extended space.

In "Space, Time and Relativity" there is the suggestion that it would be better to define the particles of matter in terms of the data of experience, and then to define space in terms of the relations between these particles.[36] Most of Whitehead's later comments on space can be conveniently considered a clarification of this assertion.

Furthermore, it is pointed out that the percipient lives in extensions and not in points; in temporal durations and not in instants. It must then be assumed that the fallacy of bifurcation was fully under fire at this early period. The implication of avoiding bifurcation is the basic conviction that science has objectively real data for investigation and not merely constructions of the human mind. Whitehead believed that Berkeley was attacking just this sort of bifurcation, but that his solution was not one which accounted for the observed facts in "Nature."[37]

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35. CN, 30.
36. STR, 124.
37. CN, 28.

Northrop, in analyzing Whitehead's physical theories, believed he had detected several instances in which that philosopher fell prey to the very procedure he had so violently condemned. There were two kinds of bifurcation, one of which was a necessary bifurcation in scientific thought, argued Northrop.[38] The first is the bifurcation between object as sensed and object as postulated, and is required in scientific reasoning. The second bifurcation <u>was</u> a fallacy, and was the one Whitehead was really attacking. This was the distinction between object and observer.

With reference to the interaction of the findings of the axiomatic method and of the inductive method, it might be noted that a certain amount of divergence between sensed object and postulated object will probably result. However, the whole aim of the axiomatic method is the selection of basic axioms and definitions so that this divergence will be abolished. With respect to the inductive method, Whitehead's assertions appear to be valid, and the most successful inductive theory will be that which describes the sensible object in terms indicated by that sensed object itself. This characteristic will reappear at frequent intervals throughout the thesis. Again, the

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38. 1941 "Whitehead's Philosophy of Science" The Philosophy of Alfred North Whitehead, The Library of Living Philosophers, 3, 206.

difficulty in reconciling the view proposed here with Northrop's lies in the question whether the sense data of experience indicate any relatedness of texture in their background. Accordingly, both species of bifurcation as listed by Northrop would be considered fallacies. In 1938 Herbert Dingle also endorsed the need for a Lockeian subject-object bifurcation.[39]

One of the difficulties in the relation of extension was on the question of its fundamental importance, and this importance was discussed by Whitehead himself in the second edition of The Principles of Natural Knowledge.[40] The appearance of the "process" motif, so important in the Whiteheadian metaphysics of 1927, was not sudden, but had developed steadily throughout the relativity writings as the problem of the final coordination of the events into a changing universe assumed more importance.

The earliest appearance of what might be considered the ancestor of process is in "Time, Space and Material," a paper which Broad has wisely declared to be a sketch of The Principles of Natural Knowledge.[41] Examining the possibilities of the various time-systems to account for what he called the "creative advance" (or "passage")[42] of

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39. "The Rational and Empirical Elements in Physics" Philosophy, 13, 148-165.
40. PNK, second edition, 202.
41. C. D. Broad. 1920 "Critical Review" Mind, new series, 29, 232.
42. TSM, 49. Cf. SMW, 152-154.

nature, Whitehead had begun to direct attention to the problems of process and its associated notion of creativity. The thing which seemed to attract him at the time was that the multiple time-systems (one for each observer) allowed a broader interpretation of this "creative advance" of nature. On this relativistic theory no duration was complete in itself; the entire set of time series was necessary for the full description of the total advance. Because of the incompleteness of each duration, no beginning or end should be assigned to any one time series, otherwise the creative advance would be precluded, he argued. This satisfied a desire for a metaphysical postulate which would exclude a beginning or an end of nature in time.

In The Principles of Natural Knowledge certain statements occur which are more representative of the notion of process: "Nature is ever originating its own development,"[43] and "one side [of nature] is development in creative advance, the essential becomingness of nature."[44] A second terminological innovation of The Concept of Nature was the appearance of the word "process" itself. Nature here was definitely conceived to be a process; it is exhibited by the fact that each duration happens and passes.[45]

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43. PNK, 14.
44. PNK, 98.
45. CN, 54.

One of the major metaphysical novelties for Whitehead's works was also the consideration of the possibility of a teleological fact implicit in nature. Victor Lowe has suggested that teleology is compatible with the works of the period, but has remarked that Whitehead's philosophy of nature makes "no attempt to assess the degree or state the kind of teleology that exists."[46] However, these sentences in The Concept of Nature seem to indicate in a vague way both the degree and the kind of teleology which would need to exist.

> The passage of nature which is only another name for the creative force of existence has no narrow ledge of definite instantaneous present within which to operate. Its operative presence which is now urging nature forward must be sought for throughout the whole, in the remotest past as well as in the narrowest breadth of any present duration. Perhaps also in the unrealized future. Perhaps also in the future which might be as the actual future which will be.[47]

Its kind, then, is apparently both efficient and final. Idealistic or theistic origins of the teleology are not considered, partly because such a development would represent a divergence from the question at hand, and partly because the relations between process and God were apparently not clear at that time. Its degree is further indicated as being great, because, when taken in conjunction

46. Victor Lowe, op. cit., 87.
47. CN, 73.

with the following statement, such a creative urge would lie at the roots of the then fundamental relation of extension.

> Nature develops, in the sense that an event e becomes part of an event e' which includes[48] (i. e. extends over) e and also extends into the futurity beyond e.
>
> An event in passing becomes part of larger events; and thus the passage of events is extension in the making.[49]

With the publication of The Principle of Relativity another fundamental change in the status of process, this time epistemological, appeared. Here process was itself available to direct experience.[50] The denial of any idealistic origin of the process, or any idealistic interference, was specifically asserted[51] in a contribution to the Aristotelian Society in the same year.

Science and the Modern World introduced another term which was fundamental in Whitehead's later works: the notion of "prehension." Rejecting "perception" and "apprehension" because of their cognitive connotations, Whitehead defined "prehension" to mean either cognitive or uncognitive apprehension. Prehensions are unifying processes

In 1925 Whitehead also introduced the idea that

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48. Note the use of the terminology of TRE.
49. PNK, 62.
50. REL, 21.
51. 1922 "The Philosophical Aspects of the Principle of Relativity" Proceedings of the Aristotelian Society, 22, 221. The letters PAPR will be used to designate this memoir.

"nature is a structure of evolving processes. The reality is the process.... The realities of nature are the prehensions in nature, that is to say, the events in nature."[52] However, "event" has here received a different interpretation from the "event" of the earlier papers; it is no longer a simple chunk of space-time in which action takes place. The "event" of 1919-1922 resembles this event of 1925 somewhat more than the earlier "event," which is closer in connotation to the relativist's "event." It should further be noticed that the mutual significance of events assumes further meaning in the notion of prehension. This insistence on "mutual significance of events" and the unifying "prehensions" again accents Whitehead's acceptance of the statement by Mach that nature does not start from simple, independent elements. This doctrine of the interdependence of the components of the natural world is one of the primary postulates of the philosophy of organism, developed as an axiomatic system in Process and Reality. A further terminological change in Science and the Modern World is the dropping of the word "passage," perhaps because it had colorless connotations which "process" did not possess.

It has been seen that for Whitehead the fundamental facts of nature are events, which are mutually significant

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52. SMW, 90.

of each other by means of the internal relatedness expressed in terms of spatio-temporal extension. Events, however, represent only one of several "modes" of diversification of nature; there are four important other modes, all of which are known as objects of different species.[53]

Perception of events must be assumed to take place internal to nature, and not as though the observer were exploring the operations of nature from without.[54] The definite name given to the perception of an event is to be known as "apprehension,"[55] an innovation of The Principles of Natural Knowledge, after the initial use of "distinguishing," as in "Space, Time and Relativity."[56]

An event whose dimensions are ideally restricted, something comparable to the classical instantaneous points, is to be known as an event-particle,[57] but has no direct perceptual significance as such. If an event-particle is a member of two events, those two events are said to intersect. When event-particles are common to the boundaries of two events, those events are said to be "in contact."[58] Furthermore, the event-particles forming the total boundary of any one event define that event uniquely.[59]

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53. PNK, 60.
54. PNK, 13.
55. PNK, 67.
56. STR, 107.
57. TSM, 48. PNK, 121. REL, 29.
58. PNK, 122.
59. PNK, 127.

An event having only three of its dimensions ideally restricted is known as a route or path;[60] if the spatial dimensions are the restricted ones, they are known as "historical routes,"[61] a term borrowed by Whitehead from C. D. Broad.

Every event is said to extend over some other event. Furthermore, there is no maximal event, since each event forms a part of a larger event.[62] In commenting on the fact that there is for Whitehead, no real lower limit, Russell suggested that such a condition may prevent the method of extensive abstraction from applying to the realm of quantum physics, where a certain minimal event is necessary for its manifestation.[63] Russell's elaboration of Whitehead's extensive abstraction provided for a finite, rather than an infinite, containing of events by a given event. This would lead to Russell's "minimal event."[64] However, Whitehead later admitted that a certain minimal duration and volume would be a necessity.

That an event is cogredient with a duration expresses the condition that that event occupies a place within that duration such that the duration is sufficient to cover it. Cogredience is thus, Whitehead argued, an immediate

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60. PNK, 124. REL, 29.
61. REL, 68.
62. PNK, 61.
63. 1927 The Analysis of Matter, 292.
64. Ibid., 288.

ingredient of experience.[65]

One of the fundamental properties of events is that they can be in only one place at any time.[66] This is to be contrasted with the classical scientific notion that a material particle can be in only one place at a time. Material particles were, for Whitehead, in space and time only derivatively, by reason of the fact that events are connected by space and time. Relations in space and time are properties of events, not objects.[67]

Miller and Gentry have criticized Whitehead's analysis of events with respect to space-time.[68] Are not events abstractions from the spatio-temporal continuum? If so, then the characters of events cannot be experienced empirically. Thus Whitehead is believed to be wrong.

In response to this criticism, it should be noted that events are indeed abstracted from space-time in the sense that they form the basis of the continuum and are parts of it. However, they are not abstractions in the sense that they form conceptual constructs from some other data of experience. Events are directly experienced. The

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65. TSM, 50.
66. PNK, 65.
67. PNK, 65. CN, 24. REL, 58.
68. David L. Miller and George V. Gentry. 1938 The Philosophy of Alfred North Whitehead, 7. This volume is filled with destructive criticism, much of which is based purely on equivocation.

spatio-temporal continuum is an expression of their relatedness. It cannot be imagined that such an assertion as made by Miller and Gentry is really damaging to the internal consistency of Whitehead's analysis, centering as it does on an equivocal use of the term "abstraction."

Events are one of the basic modes of diversification; the other four are species of objects, to be known as sense-objects, percipient objects, perceptual objects, and scientific objects.[69]

Sense-objects are the fundamental objects in nature, and are specific sense-data located in an external event, such as definite colors or tastes, and represent a type of permanence disclosed in nature.[70] The relation of object to event is that of "ingression."[71] All other objects in nature presuppose the sense-object, and there is no apprehension of nature apart from recognizing the relation of sense-object to event. As contrasted with the external event there is the percipient event which is the awareness of a polyadic relation of sense-object to nature, and expresses an awareness of the permanence to the sense-object. This permanence is to be known as the percipient object.[72] It is distinguished from the sense-object in that the

69. PNK, 60.
70. TSM, 51. PNK, 83. CN, 149.
71. CN, 144. UC, 8.
72. PNK, 83.

latter is included in the external event of nature. Although the sense-object is permanent, it is the percipient object which recognizes that permanence and expresses the relatedness of the sense-object to nature.

Perceptual objects are the ordinary common-sense things. They represent the permanence of a configuration of sense-objects throughout durations of time.[73] They might be the blueness of a chair, its softness to the touch, etc. The perceptual object is "conveyed" to the mind by the sense-object. Because the sense-object conveys the perceptual object, there is a reasonable basis for delusion on the part of a percipient.

Scientific objects are the entities conceived by science to be responsible for the physical world, and are electrons, protons, etc.[74] Any single scientific object is in some sense related to every event which contributes to the creative advance of the universe. There are the occupied events, which express the fact that the basis of the existence of the electron is "here-now." The unoccupied event expresses the fact that the field associated with the electron exerts an influence "here-now."[75] The theory of scientific objects as developed here by Whitehead

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73. TSM, 53. PNK, 88. CN, 149, REL, 54. UC, 8.
74. TSM, 55. CN, 149.
75. TSM, 55. PNK, 96, 166. The occupied event is known as "the situation of the electron" in CN, 159.

has been declared inadequate by Northrop.[76]

Perceptual objects are ordinarily "recognized" (a term for the perception of an object, similar in usage to the "apprehension" of an event)[77] as being uniform objects. That is to say, the perceptual object enduring throughout a duration A will exhibit the same characteristics in any shorter duration A_1 within the larger duration.[78] It is interesting to note that, whereas in The Principles of Natural Knowledge and The Concept of Nature the perceptual object is considered as the summation of sense object, with the appearance of The Principle of Relativity it is described as an adjective pervading its route.[79] Whitehead, however, recognized the possible objection from quantum physics that there are reasons to assume that perceptual or scientific objects are not uniform. In fact, this sort of reason was the one expressly given for allowing the existence of non-uniform objects,[80] and in his writings the extension of this notion of non-uniformity formed the basis of a tentative inclusion of quantum physics. It will be seen that in its fully mature form, it is not sufficient to account for quantum mechanics.

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76. Northrop, op. cit., 205.
77. PNK, 67.
78. TSM, 56. PNK, 167-168.
79. A divergence first noted by L. Susan Stebbing. 1926 "Professor Whitehead's Perceptual Object" Journal of Philosophy, 23, 198-199.
80. TSM, 57. PNK, 189.

Had Whitehead been as productive in searching the groundwork of quantum mechanics (for it must be remembered that relativity is the last scientific subject written about in scientific form by him), he might have admitted the insufficiency of that allowance. Non-uniform objects are held to be operative against the continuous spatio-temporal background. It is this predictable uniformity on which quantum mechanics throws a serious doubt. Sir Edmund Whittaker has pointed out that this may represent a serious objection to Whitehead's method of extensive abstraction.[81] Victor Lowe has considered the danger to be not a real one,[82] nor would any other writer who considered quantum mechanics to be explainable in terms of field theory. But it is just the point that quantum mechanics may not be thus explicable which is important. Thus, Whitehead's method of extensive abstraction is under fire from quantum mechanics along with every other scientific or philosophical theory grounded in a conviction of the fundamental uniformity and determinate continuity of nature.

In _The Principles of Natural Knowledge_ Whitehead allowed that molecules are non-uniform objects, and pointed

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81. 1948 "Alfred North Whitehead" _Obituary Notices of Fellows of the Royal Society_, 6, 295.
82. 1950 April 28 and 1950 October 8. Letters to the author.

to the fact that the essential notion behind non-uniform objects is that of rhythm. In the case of electrons and molecules the rhythm is essentially very simple, but Whitehead does not indicate what its nature might be. To solve the problem of the nature of the rhythm of the sub-atomic particles would be to solve for determinism the present deadlock in physical theory and pave the way to a possible unification.

The life-bearing object, however, is the most outstanding type of non-uniform object. Whitehead's assertion that objects can be represented in simple extensive elements of instantaneous space, and the subsequent statement that the life-bearing object needs more than an instantaneous space to assert its life-character,[83] leads to the notion of Russell that a minimal event may be necessary for a coherent account of nature.

Whitehead later specifically mentioned the possibility that minimum quanta of extensive time may be necessary for the realization of the non-uniform objects,[84] and considered the idea as in no sense inconsistent with the fact that any event must contain an infinite number of sub-events. This is possible on the grounds that the objects merely represent characters of the events, and in no way

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83. PNK, 196.
84. CN, 162.

damage the existence of the event by their contingent character.[85]

These rhythms exhibited by non-uniform objects are due to the impression of a pattern on those selected elements,[86] and the example which occurs to mind is the notion of wave-form, simple or complex. Quantum researches such as those of Dirac in 1928 lend credibility to such an interpretation, but there are many researches which are more consonant with forgetting the wave aspect completely and introducing fundamental discontinuity. The life of an individual person is an example where the pattern assumes a more complex nature, because those persons may build an additional rhythm from the aggregate of the earlier rhythms.[87] Russell, enlarging the Whiteheadian treatment of rhythms, suggested that there may be three primary types of objects, in contrast to Whitehead's uniform and non-uniform division. The occupied events would then be steady events, rhythmical events, and transactions.[88] The latter was specifically designed to describe quantum changes, and represents a more acceptable solution than Whitehead's rhythms. Russell admitted that transactions could be considered as the sudden substitution of one rhythm for

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85. This is a point on which Whitehead was accused of inconsistency by Miller and Gentry, op. cit., 40.
86. PNK, 198-199.
87. PNK, 197.
88. 1927 The Analysis of Matter, 355.

another.[89]

With the publication of Science and the Modern World, time was considered fundamentally as a succession of epochs, rather than as subject primarily to the relation of extensiveness. This was necessitated by the condition that the completion of a rhythmical pattern required a duration of a definite time-lapse, called an epoch.[90] The realization of the complete event required by the pattern thus has need of an epochal, rather than an extensive, analysis of time. The substitution of this new theory of time represents a possible advance and makes Whitehead's scientific thought more amenable to interpreting quantum mechanics.

In order, however, to explain quantum actions more successfully, it will be necessary to consider more explicitly the nature of the objects involved in them.

A uniform object, it has been said, continues unchanged throughout the exhibition of its character along its historical route. When, therefore, the uniform object is considered, it will be found to be capable of definition as a pervasive adjective,[91] since its nature pervades the entire stretch of that route. Material bodies, being adjectival to the routes of the event-particles in which

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89. Ibid., 402.
90. SMW, 157.
91. REL, 32.

they are located, cannot have the character of being the ultimate data of physics. Nor will they be the source of the spatio-temporal relation of extension, for then space-time would depend on the contingent appearance of the objects.[92] In reviewing The Principles of Natural Knowledge Theodore de Laguna singled out this assertion as unsound.[93] It is explicitly the doctrine of Einstein that the material objects will be the relata, producing one of the fundamental points of divergence between Einstein and Whitehead.

Whitehead had, therefore, frankly admitted the adjectival character of matter in the philosophy of nature period. But with respect to the question of whether material particles are really adjectival, it seems that Professor de Laguna has a highly defensible objection. By using this device, Whitehead seems to have placed his cosmology just outside the range of experiential verification, thus making precise scientific investigation at least highly improbable. For the purposes of physical science, therefore, the bifurcated cosmology of Einstein seems preferable to the unified cosmology of Whitehead at this point; the adjectival nature of matter must be postulated; it is not found. At this point, Professor van Melsen's

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92. REL, 58, 72.
93. 1920 "Review of An Enquiry Concerning the Principles of Natural Knowledge" The Philosophical Review, 29, 272.

observation that Whitehead needs something to "do the eventing"[94] becomes highly pertinent. Professor George Temple, who attempted a generalization of Whitehead's relativity, has also noted the scientific difficulty: "...the method [of extensive abstraction] comes right outside the demand of our immediate sense observations."[95]

Causality is thus an operation erroneously ascribed to objects; that property belongs only to events, which may be called "conditioning events" in their function of efficient causation. If a conditioning event is active, it is the direct cause of the occurrence of a given sense-object in its proper situation. The other events of nature are passive with respect to that sense-object, and allow it to participate in a percipient event.[96] Thus the laws of nature are a description of the operation of the active conditioning events and the percipient events; their discovery depends on the repitition of the character of the active conditioning events.[97] Hence, objects cause events only insofar as they supply the characters of antecedent events which furnish the conditions which will characterize a future event.[98] It is because a physical object must be located in an active conditioning event

94. 1950 December 4. Letter to the author.
95. 1950 July 14. Letter to the author.
96. PNK, 86.
97. PNK, 87.
98. PNK, 73.

that it is considered to be the cause. Northrop has declared this doctrine to be one of the places where Whitehead has fallen prey to bifurcation.[99] It is not altogether clear that Whitehead has escaped an object-event bifurcation here, and the difficulty is even more glaring than a possible actual entity-eternal object bifurcation in the philosophy of organism.

In reviewing The Principles of Natural Knowledge, C. D. Broad suggested that Whitehead's objects could be looked upon as analogous to the philosophical "universals"; events were the particulars.[100] However, Professor Stebbing, in discussing this particular suggested relation, has reminded those who favor the identification, of a crucial point. The notion of universals and particulars is derived from the acknowledgment of the existence of substance-quality as the basic category of physical nature.[101] It is to just this division which Whitehead was objecting under the accusation that it represents the "fallacy of misplaced concreteness."[102]

Professor Stebbing further pointed out that an

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99. Northrop, op. cit., 186.
100. 1920 "Critical Notice of Alfred North Whitehead's An Enquiry Concerning the Principles of Natural Knowledge" Mind, new series, 29, 218.
101. 1925 "Universals and Professor Whitehead's Theory of Objects" Proceedings of the Aristotelian Society, 25, 326-327.
102. SMW, 66.

identification of Whitehead's "object" with Russell's physical object as the class of its appearances is incorrect.[103] Whitehead's objects have aspects too far-reaching in their unoccupied events to admit the meaningfulness of such a classification. In 1927 Russell himself declared the divergence to exist: "... that 'aspects' may be not quite alike, and yet may be in some sense numerically one. To my mind, such a view, if taken seriously, is incompatible with science, and involves a mystic pantheism."[104]

Professor Weiss, criticizing Whitehead's notion of the passage of nature and its connection with objects, believed he had found a discrepancy. If things are but momentary adjectives of events, then even the oldest of objects must be constantly new.[105] In response to this challenge it must be pointed out that there is a persistence of the same object throughout the route it pervades. The newness of the object is really the newness of the event; the newness is due to the fact that the characters describing the old object are located in a new conditioned event. Nevertheless, it does not seem that Whitehead accounts for persistence in nature as easily as he accounts

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103. L. Susan Stebbing, _op. cit._, 308.
104. _The Analysis of Matter_, 340-341.
105. Paul Weiss. 1936 "The Nature and Status of Time and Passage" _Philosophical Essays for Alfred North Whitehead_, 172.

for change. This is a criticism which will continue in the next part of the thesis on the philosophy of organism.

In connection with the persistence of an object, it is well to note an innovation in Science and the Modern World. There an "eternal object" is introduced.[106] A color, for example, is an eternal object. Eternal objects are thus recognizable without reference to any one particular event. An eternal object has, however, a particular method of ingression into that event without losing its individuality. Using language with a definitely metaphysical flavor, Whitehead said, "each eternal object is an individual which, in its own peculiar fashion, is what it is. This particular individuality is the individual essence of the object...."[107] An eternal object was also described as having the metaphysical status of "a possibility for an actuality."[108] It is certain, however, that Whitehead did not imply thereby the problems associated with the problem of universal-particular. Furthermore, eternal objects were described as being related by the term "ingression" to what was now an "actual occasion." It must therefore be necessary to assume that sense-objects have their roots in the eternal object. Apparently it is through the route of the sense-object that an eternal

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106. SMW, 107, 197.
107. SMW, 197.
108. SMW, 198.

object has its ingression into an event.

On two grounds, A. O. Lovejoy found the eternal objects to be undesirable components in a nature philosophy.[109] In the first place, Lovejoy denied that an eternal object which is ingredient in an event can remain the same after its ingression. The difficulty carries over into the philosophy of organism, and a suggestion for the possible improvement of the situation will be made in Chapter X. The second ground for Lovejoy's objection was that if eternal objects were timeless, they again should have no proper function in nature. This argument, it seems, should be tempered to the point of making it an assertion of unverifiability. The revised argument then joins those questioning the experimental value of Whitehead's cosmology. In its original form, a much more debatable ontological question is invoked.

Santayana has found Whitehead's treatment of sense-objects and scientific objects reminiscent of his own essences, and has used the definition of an eternal object from _Science and the Modern World_ as a documentation of the fact that essences had been clearly distinguished. Santayana further opined that Whitehead did not call them essences because of the circumstances in which he approached

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109. 1930 _The Revolt Against Dualism_, 111.

metaphysics: that of English realism of the 1920's.[110]

By examining the fundamental relation of extension expressed in the symbolic form a K b,[111] Whitehead introduced his method of extensive abstraction proper. As in the 1905 memoir, Whitehead seemed to be searching for the "forms in the facts," as did Plato. The method centered primarily around the use of what he called abstractive sets or classes.[112] These classes had the property of having each of its members extended over, and was itself extended over by, other members of the set. Furthermore, there was to be no smallest member of the set. However, inasmuch as certain boundaries of certain types of events were clearly defined so that the event was of a certain size, exceptional abstractive classes must exist which converge to the boundaries of the given event rather than to an extensionless, ideally restricted member of the ordinary abstractive class.[113]

In terms of extensive abstraction, the moment was defined to be that minimum duration which could extend over elements of all types of abstractive sets: spatial or temporal.[114] This definition gives rise to the existence of

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110. George Santayana. 1928 The Realm of Essence: Book First of Realms of Being, 169-171.
111. Read, "a extends over b."
112. PNK, 104.
113. PNK, 107-108.
114. PNK, 109-111. CN, 57.

many alternative time-systems, each of which was capable of possessing its own peculiar spatial system, which for that time-series was absolute.[115] This accounts for the fact that an individual does, in fact, use one determinate measure system.[116] Moments in the same time-system were defined to be parallel; those which were in different time-systems were accordingly non-parallel. When moments of different time-systems intersected, different spatial elements called levels, rects, and puncts could be defined, and would be analogous to planes, lines, and points of the ordinary three-dimensional Euclidean space.[117]

Using a technique inspired by Wilson and Lewis,[118] Whitehead defined congruence in terms of parallelism. The particular novelty of the treatment was that not only spatial, but also temporal, congruence was uniquely defined.[119]

Because of the fundamental nature of moments in a time-system, spatial order in one time-system was defined, and derivatively coordinated with the spatial order in other time-systems.[120] The assumption that the spatial

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115. CN, 106.
116. Cf. STR, 126.
117. PNK, 116. CN, 91-92.
118. Edwin B. Wilson and Gilbert N. Lewis. 1913 "The Space-Time Manifold of Relativity. The Non-Euclidean Geometry of Mechanics and Electromagnetics" Proceedings of the American Academy of Arts and Sciences, 389-507.
119. PNK, 141. CN, 137. REL, 52.
120. PNK, 119. REL, 60.

order of two time-systems are extremely closely coordinated accounts for the fact that, in the main, the measure systems of different individuals appear to agree within the limits of observation.[121] It must be remembered that this method of extensive abstraction depends finally upon the uniform relatedness exhibited as being the texture of nature. At this point of development, James Johnstone called a halt because of his conviction that the moments of nature did not exhibit a uniformity of passage, and therefore were not amenable to the method.[122] This, of course, is a point of fundamental divergence, and is more consistent with Einstein's method (as Johnstone has offered it), as well as foreshadowing Whitehead's own later epochal theory of time.

With respect to the fundamental relation of extension, Theodore de Laguna offered the suggestion that the notion of "containing" could be substituted, and the method would thereby gain in simplicity.[123]

A further novelty of extensive abstraction was one due primarily to Bertrand Russell, and was known as "partial overlapping."[124] By the time that the notion appeared

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121. Cf. STR, 126.
122. 1922 "On the Limitations of a Knowledge of Nature" Proceedings of the Aristotelian Society, 22, 48.
123. 1921 "Extensive Abstraction: A Suggestion" The Philosophical Review, 30, 216.
124. 1927 The Analysis of Matter, 294.

from Russell's hand, Whitehead had finished the works of this period. Even Russell did not carry the investigation further because of its limited value in physics. He suggested, however, that it may prove useful in psychology.

Miller and Gentry have suggested that there exists an ambiguity in this stage of Whitehead's development concerning abstract methods, logical concepts, and potentiality, as in the case of the ingression of sense-objects into events.[125] This charge seems defensible, and perhaps was one of the reasons for the adoption of process as the fundamental reality in nature with the appearance of Science and the Modern World, and for its final refinement in Process and Reality.

In Whitehead's treatment of the spatio-temporal continuum, another of the desirable consequences was that passage along a spatial route has a different character from passage along a historical route, thus giving a somewhat different status to time and space.[126] This, as has been seen, was one of the philosophical disadvantages of many of the relativity theories, where space and time were completely homogeneous.

One of the characteristics of a particle pursuing a purely historical route (at rest) in one time-system, is

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125. 1938 The Philosophy of Alfred North Whitehead, 33.
126. REL, 68.

that the same apparent particle will be moving in another time-system with uniform velocity.[127] A further consequence of the nature of durations is related to this situation. Inasmuch as a duration is immediately tied to one spatio-temporal system, it would of necessity fail to include all the aspects of nature in other spatio-temporal systems. Accordingly, no single duration can be conceived to represent the whole of nature without any omissions; nature is larger than any one of the spatio-temporal systems contributing to its entirety.[128]

One of the convictions Whitehead upheld, which was closely associated with the problems of relativity, was the belief in the objective meaning of simultaneity. This disagreement formed a third major area of contention with the orthodox relativists. The first two areas, it will be remembered, center around the homogeneity of space-time and the status of material particles as substantial or adjectival to events.

Such a statement is made possible by the fact that the elements which may lay claim to simultaneity have the common property of being components of the same duration. Accordingly, there is an empirical basis for the objective simultaneity of two events.[129] The solution of the

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127. REL, 70.
128. PNK, 81.
129. CN, 56.

problem, however, of establishing the simultaneous reception of one given event by two observers, i. e., the establishment of a public time, is not apparent in Whitehead's writings. It was specifically on this point that Northrop criticized Whitehead, and quoted a statement arising in an interview with Einstein, who was of the same opinion.[130] There is no reason to assume that the events cogredient with the duration A in time-system α are going to be cogredient with the duration B in time-system β. The solution in The Concept of Nature is far from satisfactory:

> For two minds, the discerned components of the general facts exhibited in their respective acts of sense-awareness must be different. For each mind, in its awareness of nature is aware of a certain complex of related natural entities in their relations to the living body as a focus. But the associated durations may be identical. Here we are touching on that character of the passage [of] nature which issues in the spatial relations of simultaneous bodies. This possible identity of the durations in the case of the sense-awareness of distinct minds is what binds into one nature the private experiences of sentient beings. We are here considering the spatial side of the passage of nature. Passage in this aspect of it also seems to extend beyond nature to mind.[131]

Here there is postulated the possible identity of the durations of two time-systems. However, such a postulate is by no means warranted by the rest of the treatment of the time-series. It seems scarcely possible to postulate

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130. Northrop, op. cit., 200-204.
131. CN, 55-56.

public time merely on the basis of a contingent identification which is not intrinsic to the structure of the analysis. The introduction of the relationship of time to "mind" does not help solve the problem. Ushenko has roundly criticized Northrop on the grounds of misrepresenting Whitehead to Einstein.[132] However, Ushenko does not himself consider the problem of how Whitehead does account for public time, and cannot really be considered to have demonstrated Northrop incorrect. The opinion presented here concurs with Northrop against Ushenko on the criticism that there is not an adequate basis for accounting for the existence of a public time.

As the writings of this period progress, there is an increasing reference to the role of mind as an ingredient in nature. In 1916, Whitehead's reference to mind occurred in the comment that the world of scientific thought was a world of ideas, of relationships between abstract concepts.[133] As to the dependence of nature on mind, Whitehead was not prepared to assert a positive relation between the two; surely it was not of such importance to warrant attention.

Two years later, in the Preface to The Principles of

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132. 1949 "Einstein's Influence on Contemporary Philosophy" Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, 7, 625.
133. OT, 61.

Natural Knowledge, Whitehead explicitly denied the value of bringing mind into the natural picture: "None of our perplexities as to Nature will be solved by having recourse to the consideration that there is a mind knowing it."[134] In The Concept of Nature the swing towards the inclusion of mind had begun to manifest itself directly: "I am not denying that there are relations of natural entities to mind or minds other than being the termini of the sense-awareness of minds."[135] Further, Whitehead proffered the possibility that "The values of nature are perhaps the key to the metaphysical synthesis of existence."[136] Statements of this nature, as well as the other innovations already noted easily explain the readiness on the part of many philosophers to welcome Whitehead as a significant philosopher.

The publications in the year 1922 exhibit essentially two viewpoints which are somewhat inconsistent, and may possibly be an indication of the conflict in Whitehead's own mind as to the significance and degree of involvement of mind with nature. Each of the three short memoirs of that year reject the mutual significance of mind and nature. In February: "...we know of events whose connexion with any mental process, as we know it, appears to be

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134. PNK, vii.
135. CN, 5.
136. CN, 5.

doubtful, incomplete, and extremely unessential to them. That is my reason for being very shy of leaning too heavily on mind in any endeavour to express the general character of reality."[137] In July, Whitehead declared, "There is a process of nature which is obstinately indifferent to mind."[138] In November, in the Presidential address to the Aristotelian Society, the position was clarified: "I differ from the idealists, so far as they consider such an external significance [that of the factors of fact] as peculiar to consciousness and thence deduce that the things signified have a peculiar dependence on consciousness."[139] It is possible that Whitehead was protesting primarily against the value for prediction of a nature-mind relation, and not against the existence of any such relations. In the James-Scott Prize Lecture to the Royal Society of Edinburgh in June of that year, Whitehead declared, "I entirely agree that the factors of nature are also significant of factors which are not included in nature."[140] With the appearance of <u>Science and the Modern World</u>, Whitehead advised, "So far as concerns what will be said

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137. 1922 "Discussion: The Idealistic Interpretation of Einstein's Theory" <u>Proceedings of the Aristotelian Society</u>, <u>22</u>, 131.
138. PAPR, 221.
139. UC, 12.
140. REL, 21.

in these lectures, your ultimate outlook may be realistic or idealistic,"[141] but he chose for himself what he called a "provisional realism." However, the swing toward including mind as one important fact had manifested itself: in the psychological field, "the mind is the major permanence, permeating the complete field, whose endurance is the living soul."[142]

The pages of Science and the Modern World contain the only discussion of the relation of God to the natural system that Whitehead wrote during the period under consideration. In the synthesis of an actual occasion, the various eternal objects achieve a complex unification, and include the relations which produce "memory, anticipation, imagination, and thought."[143] The principle which is responsible for this process of the synthesizing of a complete actual occasion is known as the Principle of Concretion,[144] which performs a task analogous to that of the Prime Mover in the Aristotelian metaphysics. The Principle of Concretion thus represents a part of the nature of God, along with the Principle of Limitation. This other principle, in conjunction with the Principle of Concretion, performs the selections from among the possible components of a

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141. SMW, 80.
142. SMW, 251.
143. SMW, 212.
144. SMW, 216.

complete actual occasion. In making these selections some limitations on the realm of possibility is necessary, and God provides the grounds for this process of limitation.[145] This function is analogous to God's primordial conceptual valuation of the eternal objects in the philosophy of organism. Because of the imposition of these limitations contributing to the concretion of actual occasions, reasoning is made possible, and value emerges with an objective reality. God thus provides both the ground of concreteness and the limitations which form the actual occasions.

In the evolution of Whitehead's philosophy of nature, and in the development of the relativity writings, it has become apparent that the earlier treatment given in "On Mathematical Concepts of the Material World" has become obsolete. But the method of that memoir has begun to assume increasing importance.

With respect to the changes necessary in the light of these further discussions, the status of time, for example, must be included in the essential relation itself. Furthermore, as much doubt has been cast on considering rectilinear extensions in Euclidean space as upon the points of space and instants of time, as having a reasonable claim to being the ultimate existents in nature. Instead, events have become the "ultimate existents" for Whitehead,

145. SMW, 221.

and extension, the "essential Relation."

Accordingly, in preparing a concise treatment in the style of the 1905 memoir, certain basic revisions are necessary for a possible Case III.

The essential relation will be that of extension, and will be defined by the statement a K b, following Whitehead. Ultimate existents will be four-dimensional events. A tentative treatment of the set of axioms might then follow.

I $\text{Hp } R . \equiv . \exists ! a$ Df

II $\text{Hp } R . \equiv . (\exists x)(a \text{ K } x)$ Df

III $\text{Hp } R . \equiv : (a,b) : a \text{ K } b . \supset . a \neq b$ Df

IV $\text{Hp } R . \equiv :. (a,b) :. a \text{ K } b : \supset : (\exists c,d) [c \text{ K } a . b \text{ K } d . c \neq a . b \neq d]$ Df

V $\text{Hp } R . \equiv :. (a,b,c,d,m,n) : b \text{ K } (c,d,\ldots,m) . a \text{ K } (c,d,\ldots,m,\ldots n) : \supset . a \text{ K } b$ Df

VI $\text{Hp } R . \equiv : (a,b,c) : a \text{ K } b . b \text{ K } c : \supset . a \text{ K } c$ Df

VII $\text{Hp } R . \equiv : (a,c) : a \text{ K } c : \supset : (\exists b)(a \text{ K } b . b \text{ K } c)$ Df

VIII $\text{Hp } R . \equiv : (\exists c)(c \text{ K } a . c \text{ K } b)$ Df

IX $\text{Hp } R . \equiv . \text{Dim}_k' a = 4$ Df

Then, defining the moment by means of the method of extensive abstraction, and symbolizing it by annexing the subscript k to the letter denoting the abstractive element, moments of the time system α will be denoted by $a_{k\alpha}$, $b_{k\alpha}$, $c_{k\alpha}$, etc. It will furthermore be true that

1.01 $\vdash : a_{k\alpha} \parallel b_{k\alpha} \parallel c_{k\alpha} \parallel \ldots$ Df

$$\text{X Hp R.} \equiv : (a_{k\alpha}, a_{k\beta}) : a_{k\alpha} \cap a_{k\beta} = lvl_{k\alpha} = lvl_{k\beta} \quad \text{Df}$$

$$\text{XI Hp R.} \equiv : (a_{k\alpha}, a_{k\beta}, a_{k\gamma}) . a_{k\alpha} \cap a_{k\beta} \cap a_{k\gamma} = rct_{k\alpha} = rct_{k\beta} = rct_{k\gamma} \quad \text{Df}$$

$$\text{XII Hp R.} \equiv : (a_{k\alpha}, a_{k\beta}, a_{k\gamma}, a_{k\delta}) . a_{k\alpha} \cap a_{k\beta} \cap a_{k\gamma} \cap a_{k\delta} = pct_{k\alpha} = pct_{k\beta} = pct_{k\gamma} = pct_{k\delta} \quad \text{Df}$$

XIII, XIV, and XV Hps R will establish the punct-order of α-puncts on α-rects, following the technique of the 1905 memoir. XVI Hp R will establish the sequence of parallel moments in the time-systems. XVII Hp R establishes the continuity of abstractive elements in the extensive continuum.

There will then be a series of axioms establishing the fact that certain types of objects ingress into given abstractive elements. However, a difficulty is encountered with respect to assigning minimum limits to the exceptional abstractive classes which manifest the characters of non-uniform objects.

Furthermore, additional axioms will be required to establish the geometrical relations developing from the betweenness relation of the 1905 memoir. This can be done fairly straightforwardly, using puncts. A relating of puncts to points must also be done definitionally.

Just how the important principles of concretion and limitation are to be symbolically expressed is far from clear.

As an extraneous relation, a single kinetic axis frame will be necessary to describe motion.

Definitional attempts must then be made to related this axiomatic system to the described components in the external world or to sensed phenomena, depending upon the outlook adopted.

Such a solution as outlined above would apparently overcome the difficulties of the 1905 memoir with respect to the innovations underlined by Whitehead and by the relativistic cosmologies, but it would be subject to the objections which have been lodged against the method of extensive abstraction.

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CHAPTER SIX

WHITEHEAD'S THEORY OF RELATIVITY

Whitehead, having finished his collaboration with Russell on the first three volumes of Principia Mathematica, turned his attention to the problems concerning the relations of geometry to space. This was a natural interest, since the fourth volume of Principia was planned to cover the problems of the foundations of geometry and would likely have considered their relation to physical space. (See page 84 of this thesis.)

Probably the first attempt in this direction which was published was "La Theorie Relationniste de l'Espace" which discussed the various kinds of space: physical, apparent, and abstract.[1] The problem was apparently deflected into channels leading to relativity, because, in reply to a question by Victor Lowe, Whitehead remembered that what bothered him at that time was "the muddle geometry had gotten into" in relation to the physical world.[2]

Whitehead was convinced that relativity theory was at least heading in the proper direction for a solution, for

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1. TRE, 423-425.
2. Quoted in Victor Lowe, op. cit., 66.

in 1916 he admitted that it represented a reasonable and simple solution to the problem of handling relative motion and for taking into account the fact that physical effects required time for their transmission.[3]

Convinced that relativity theory only weakened, and did not remove, the bifurcation of nature involved in the classical theory,[4] Whitehead proceeded to provide an empirical basis for a more coherent account of nature which would avoid bifurcation. His writings of this period, because of their philosophical interest, spread the interest in relativity to philosophers, who might otherwise have continued oblivious to the revolution which had taken place in science, or avoided it as long as possible. McGilvary went so far as to hold Whitehead responsible for the philosophical interest.[5] It is certainly true, however, that Eddington and Russell must have played a large part in drawing sympathetic interest to the theories.

Whitehead repeatedly acknowledged indebtedness to Einstein and Minkowski for the invention of the theory, despite the fact that he had constantly rebelled against the interpretation placed on the new theory by Einstein

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3. STR, 127-128.
4. CN, 41.
5. Evander Bradley McGilvary. 1941 "Space-Time, Simple Location, and Prehension" The Philosophy of Alfred North Whitehead, The Library of Living Philosophers, 3, 212.

and the orthodox relativists.[6] In an article written for the London Times, Whitehead declared,

> Einstein's work may be analyzed into three factors--a principle, a procedure, and an explanation. The discovery of the principle and the procedure constitute an epoch in science. I venture, however, to think that the explanation is faulty.[7]

Perhaps one of the two fundamental points of divergence between Whitehead's relativity and that of the orthodox relativists draws its roots from Whitehead's preoccupation with the epistemological problem involved. Whitehead's epistemological solution derived its answer from the fact that events were mutually significant of each other, and therefore the spatio-temporal continuum must be uniform, in contrast to Einstein's demand for non-uniform curvature in the general theory. This uniformity was made possible by the internal relatedness of events in contrast to the external relatedness of general relativity. It is odd that probably no other philosopher has taken up the challenge on the point of external relatedness in orthodox relativity. Perhaps it never occurred because it was not emphasized in the relativistic writings. Thus, on Whitehead's theory, it becomes possible to infer the nature

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6. STR, 126. PNK, vi. REL, 88.
7. 1920 February 12 "Einstein's Theory: An Alternative Suggestion" The Times Educational Supplement, No. 252, 83. This paper will hereafter be referred to as ETAS.

of the whole of space-time on the basis of a small part of the continuum.

On Einstein's theory, however, the inference is impossible. For in orthodox relativity, the nature of space-time depends upon the contingent appearance of material bodies. Thus no accurate appraisal of an entire spatio-temporal continuum is possible until the location of every material particle in the universe is known. The situation is rendered even more difficult in the unified field theories based on a parallel transport operation.

Russell's comment on the position was that, essentially, neither side had proved its case, and that he preferred the "variable" geometry of Einstein on the grounds of comprehensiveness,[8] although it will be seen that Whitehead's theory seems equally comprehensive.

Whitehead was therefore fully in favor of a flat space-time, which might be chosen because of its simplicity.[9] He was prepared to accept the notion that the permanent space for each time-system was uniformly elliptic or hyperbolic if need should be shown for it.[10]

He further explicitly rejected the need for a space-time manifold of more dimensions that found acceptance in many theories. The desire for a four-dimensional continuum

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8. 1927 <u>The Analysis of Matter</u>, 79-80.
9. REL, 76.
10. REL, v.

was preferred for epistemological reasons.[11] He was of the opinion, however, that it was an indication of an attempt to reintroduce uniformity that some writers had increased the number of dimensions to five.[12] This suggestion, coupled with Professor Wilson's "Body Alpha," seems to imply that the classical notions of an absolute Euclidean space-time were so firmly rooted that even the relativists were loathe to relinquish them entirely.

By use of his method of extensive abstraction and a definition of congruence borrowed from Wilson and Lewis,[13] Whitehead developed the mechanics of a "moment," considered as an event-particle. Introducing an arbitrary velocity-squared constant _k_ instead of Lorentz's light-velocity _c_, Whitehead demonstrated three types of kinematics possible: hyperbolic, elliptic, and parabolic. In the hyperbolic type, $k = c^2$, and the formulae governing the Fitzgerald-Lorentz transformations follow.[14] Elliptic kinematics require that $k = -h^2$, where h is an arbitrary constant. This type, however, makes space and time completely homogeneous, and the world of events is not easily intelligible with such a homogeneity. The parabolic kinematics, using $k = \infty$, presented the limiting case of Newtonian

11. ETAS, 83.
12. ETAS, 83.
13. PNK, 141.
14. PNK, 159.

mechanics. Accordingly, it is the hyperbolic mechanical system which is required. In this respect, Whitehead parallels Lorentz and Einstein.

One divergence arises, however, in the interpretation of the meaning of the velocity c. Whitehead repeatedly refused to allow it to be interpreted as meaning the velocity of light *in vacuo*.[15] This, he insisted, was an arbitrary designation, and placed a greater burden on a non-sensible quantity than it can carry.

Investigating the possible treatment of an event, Whitehead observed that essentially it could be handled as a field because of its internal relatedness to all other events. Therefore it would be possible to express the laws of the physical field in the tensor form utilized by Einstein. However, the tensor formulae will be different, inasmuch as the Einstein equations demanded that the physical laws be independent of the circumstances in which their measurements are made. Whitehead, on the other hand, suggested that the demand was arbitrary, and that those very circumstances might affect the measurement itself.[16]

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15. PNK, 53-54. CN, 195. REL, 240. SIM, 39. ETAS, 83.
16. 1924 Notes recorded by the Secretary on Whitehead's comments in the discussion after the reading of the memoir by George Temple. 1924 "A Generalisation of Professor Whitehead's Theory of Relativity" *Proceedings of the Physical Society of London*, *36*, 193. Apparently no other commentator on Whitehead has noticed this short, but informative, record.

Considering a stretch of the historical route of an adjectival particle, Whitehead included an account of both the gravitational and the electromagnetic fields.[17] Here is another essential innovation by Whitehead: a stretch of the route traced by an event-particle rather than an idealized infinitesimal element of that route is the basic unit for the formula. The total impetus of the combined fields is described by the formula,

$$dI = M\, dJ^2 + \frac{E\, dF}{c} ,$$

where M is the mass of the particle, dJ is the homogeneous first degree expression for the mass-impetus of M along the historical route, E is the charge in electrostatic units, dF is the homogeneous first degree expression for the electromagnetic impetus along the historical route.[18]

This adoption of a purely physical quantity, the "impetus," clearly followed the warning of Poynting, who held that physical science must describe the sensible in terms of the sensible. It represents perhaps the second of the major divergences with Einstein. The impetus was a quantity which should be more attractive to experimental physicists than the metric element of Einstein. But, as with Professor Temple's later contribution in the same

17. REL, 78-79.
18. REL, 79-80.

tradition, most of the relativistic physicists seem to have bypassed it. But this device, which seems allowable, avoids the physical-geometrical bifurcation which troubled Einstein after the introduction of general relativity. It avoids the bifurcation even more than do the new unified field theories, inasmuch as the physical world is still described by unified physical, as opposed to unified geometrical, concepts.

The empirical facts expressing a pure gravitational field are then given by an equation,[19]

$$dJ^2 = dG_M^2 - \frac{2}{c^2} \sum_m \psi_m \, dG_m^2 \quad ,$$

corresponding to Einstein's expression for ds^2. An adjectival mass-particle in the kinematic element is represented by m, and ψ_m represents the gravitational law of diminishing intensity. The factor $2/c^2$ accommodates the expression to the empirical law of gravitation. The dG's are the invariant expressions for the quantitative phenomena of the advances of the analogous historical passages of m and M. The expression ψ_m, defining the diminishing potential of the gravitational field, might be known by the name given it by Broad, a "retarded potential."[20] Whitehead criticized Einstein for misinterpreting

19. REL, 81.
20. C. D. Broad. 1923 "Critical Notice: A. N. Whitehead's Principle of Relativity" Mind, new series, 32, 219.

the dJ^2 to be a spatio-temporal element. Einstein's hesitation on this point is shown by the fact that he admitted it to be the reason for waiting seven years after finding the full general relativity theory before publishing it. Einstein was then convinced that the gravitational expression did coincide with the metric of space-time; Whitehead was not. Again the divergence accents the two major areas of disagreement.

The two parts of the J's are known as the "potential" (roughly analogous to the gravitational potential) and the "associate potential."[21]

An advantage deriving from Whitehead's various time-stratifications (which give rise to the difficulty of not adequately explaining public time) is that four alternative renderings of a gravitational law in terms of relativity are possible. Of these four, one is the form suggested by Einstein and another is the form given above by Whitehead. The other two are roughly analogous to Einstein's, but are more complex and are not linear. One of the other two alternatives contains a law which might be fitted to account for an interaction of the electromagnetic and the gravitational fields and introduces a contravariant tensor $T^{\mu\nu}$ to describe the operation of the electromagnetic field.[22]

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21. CN, 183. ETAS, 83.
22. REL, 86-87.

On the question of the experimental confirmation of the law for dJ^2, Whitehead's theory is somewhat closer to observation than Einstein's. Unfortunately, certain phenomena considered crucial to confirmation of Einstein's theory are equally predicted by the flat space-time solution of Whitehead, and may represent as serious an objection to Whitehead's flat space-time solution as to Einstein's.

With respect to the advance of Mercury's perihelion, Whitehead's theory provides the identical additional term for the advance of Mercury's perihelion, as well as the additional terms for the other planets, which, in the case of Venus, is an undesirable addition.[23]

The deviation of a light ray by a gravitational field also follows directly,[24] and is subject to the same observational uncertainty.

With respect to the spectral shift of lines in the sun, Whitehead is on safer ground. If the assumption be made that the static distribution of the electric charges in an atom determine its cohesive forces, Einstein's predicted shift would be increased by one-sixth.[25] However, assuming that the internal forces of a molecule are not affected by a gravitational field, a result of about

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23. REL, 105.
24. REL, 110-111.
25. REL, 102-103.

two- to three-fifths that of the spectral shift required by Einstein will be predicted. Furthermore, there will be a broadening of the lines.[26] Whitehead's strong point is that the molecules are postulated to be oriented in every possible manner,[27] and that therefore highly discrepant results may follow. This explains in a more natural manner the observations on the red-shift.

A further effect predicted by Whitehead's theory, subject to alteration when more is known of subatomic processes, is that there will be a further red-shift in lines on the limb of the sun with respect to those at the sun's center.[28] This prediction seems to be confirmed by experimental verification when considering the molecule to be a vibrating doublet.

In the case of a nebula, some doubling or even trebling of spectral lines is predicted, due to the different orientations of the light-emitting molecules.[29] This prediction is, as the preceding two predictions, subject to confirmation of the assumption regarding a vibrating molecule.

With respect to the motion of the moon, Einstein's theory is only partially able to deal with certain

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26. ETAS, 83.
27. REL, 115.
28. REL, 117-120.
29. REL, 121-126.

irregularities of its motion. Whitehead's theory, allowing for varying gravitational potentials, can lay a claim to attempting an explanation, although it may conceivably fail as well. The problem was considered by Whitehead, but because a period of about 250 years was necessary to obtain a good degree of prediction, he did not advance any definite predictions or claim any empirical comparison with results in the Lunar Tables.[30] Of course, Eddington's statement of doubt regarding an ability of a gravitational theory to explain the moon's irregularities may be equally powerful here.

Considering the effect on a gravitational field of the temperature of the attracted body, Whitehead found that the correcting term for ordinary molecular velocities was inappreciable.[31] If, however, subatomic velocities, which may conceivably be extremely large, are taken into account, the temperature correction may be appreciable.

It has been mentioned that Whitehead considered the possibilities of the interaction of the gravitational and electromagnetic fields. However, no attempt was made to represent the electromagnetic field by a skew-symmetric tensor, as Einstein and others have done. As with the

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30. REL, 84, 132-136.
31. REL, 113.

gravitational field, the electromagnetic field would need to be described, in Whitehead's theory, on an empirical basis.

The alternative form of the gravitational expression suggested by Whitehead would possibly lead no further, however, than current unified field theories. A worthwhile subject for further investigation would be the elaboration of this alternative law in order to see if any definitive predictions might be made which would recommend its serious consideration or legislate against its acceptance.

Judging from the treatment of the laws of the physical field considered here, it is well to remember that Whitehead's alternative theory of gravitational fields has at least as much to commend itself to the scrutiny of the physicist as any current relativity theory. It is particularly attractive from the point of view of the epistemologist, for his problems have by no means been solved by the current relativistic theories. Furthermore, it is possible to provide a straightforward metaphysical structure within which this model of the universe can operate.

Professor George Temple is apparently the only member of the scientific world who has attempted to consider seriously Whitehead's relativity. In a memoir offered to the Physical Society of London early in 1924, Professor

Temple offered a generalization of the 1922 theory offered by Whitehead. By introducing the substitution of a space-time manifold of uniform and isotropic curvature, Temple showed that the deviation of the light rays in a solar gravitational field would be deflected by an inappreciable amount--2×10^{-8} of the Einstein value of 1.74".[32] This result would, of course, throw the entire problem of the deviation of light rays open to investigation again. In the present state of empirical evidence, however, this reopening of the problem may be justified. A further conclusion of Professor Temple's memoir was that a space of positive curvature possessing the radius of minimal value of 10^{16} km. would be associated with Whitehead's cosmological model in "curved" space-time.[33]

The substitution of this space of uniform positive curvature would bring Whitehead's model more into consonance with recent investigations. There seems to be no reason why Whitehead's cosmological model should not become a member of the kinematic family. The prejudice against the possibility of the existence of a unique time-origin at $t = 0$ would, however, need to be removed. With the introduction of the epochal theory of time, the

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32. George Temple. 1924 "A Generalisation of Professor Whitehead's Theory of Relativity" <u>Proceedings of the Physical Society of London</u>, <u>36</u>, 191.
33. <u>Ibid.</u>, 191.

prejudice would seem capable of being abolished. A change (with respect to the possibility of the existence of a maximum event which would represent all of nature in one time-system) in the abstractive sets would be necessitated if the epochal theory were not assumed. Whitehead's time in the old extensive theory closely resembles Milne's τ-time.

In Whitehead's theory, the creation of new matter would not be a point of inconsistency,on the grounds that new adjectival particles would not appear without the proper active conditioning events to provide them. There is the possibility that matter would be capable of creation in time on the grounds that, in the act of the concretion of an actual occasion, the eternal objects may conceivably provide a new configuration which would be responsible for the creation of new adjectival particles. At any rate, creation of new matter would not represent a challenge as serious to Whitehead's relativistic model as to Milne's. The objection to considering matter as adjectival remains, however.

The conclusion of this chapter, then, is that the Whiteheadian relativity theory may well repay further attention from the physicists as having a claim to reliability and comprehensiveness. It seems that Professor Temple's generalization utilizing space-time of uniform positive curvature is preferable in view of the present state of empirical evidence. A fuller explanation of the phenomenon of the apparent displacement of a light ray in a gravitational field may possibly be expected in considering the full effects of the electromagnetic impetus. It may be that the prejudice against a uniform space-time is not so well-grounded as is presumed by many relativists. In support of this argument, a recent observation by Professor Temple may be cited:

> I still think that Whitehead's alternative relativity theory has a great deal to recommend it, and I am certainly not prepared to agree that it has any less validity than the theories put forward by Einstein.[34]

Whitehead had then presented a complete cosmological theory in the period immediately after the introduction of relativity. In considering its metaphysical meaning, however, he found it necessary to provide more alterations because of the evolution of the concept of process as being at the roots of reality. Whitehead's serious purely scientific writing was at an end; metaphysics now assumed the central preoccupation. But his was a metaphysics grounded in the physical sciences, and he had yet many suggestions to offer to cosmology which were to be perhaps among the most important additions to cosmology since the rise of the scientific method.

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34. 1950 July 14. Letter to the author.

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